

Lecture 4  
2020/2021

# Microwave Devices and Circuits for Radiocommunications

# 2020/2021

- 2C/1L, MDCR
- Attendance at minimum 7 sessions (course + laboratory)
- Lectures- associate professor Radu Damian
  - Wednesday 15-17, Online, Microsoft Teams
  - E – 50% final grade
  - problems + (2p atten. lect.) + (3 tests) + (bonus activity)
    - 3p=+0.5p
  - all materials/equipments authorized

# 2020/2021

- Laboratory – associate professor Radu Damian
  - Thursday **10-12**, II.13 / (**Online**)
  - L – 25% final grade
    - ADS, 4 sessions
    - Attendance + personal results
  - P – 25% final grade
    - ADS, 2 sessions (**-1~25.02.2021**)
    - personal homework

# Materials

- RF-OPTO
  - <http://rf-opto.eti.tuiasi.ro>
- **David Pozar, “Microwave Engineering”,**  
Wiley; 4th edition , 2011
  - 1 exam problem ← Pozar
- Photos
  - sent by ~~email~~/online exam
  - used at lectures/laboratory

# Profile photo

- Profile photo – online “exam”

**Examene online: 2020/2021**

**Disciplina: MDC (Microwave Devices and Circuits (Engleza))**

**Pas 3**

Nr.	Titlu	Start	Stop	Text
1	Profile photos	03/03/2021; 10:00	08/04/2021; 08:00	Online "exam" created f ..
2	Mini Test 1 (lecture 2)	03/03/2021; 15:35	03/03/2021; 15:50	The current test consis ..

# Online

- access to **online exams** requires the **password** received by email

English | Romana |

Main Courses Master Staff Research **Student List**

Grades Student List Exams Photos

## POPESCU GOPO ION

Fotografia nu există

Date:

Grupa	5700 (2019/2020)
Specializarea	Inginerie electronica si telecomunicatii
Marca	7000000

[Access the site as this student](#) | [Request access to software](#)

**Grades**

Inca nu a fost notat.

Main Courses Master Staff Research

Grades **Student List** Exams Photos

### Login

Use the last name and email stored in the database

Name  
POPESCU GOPO

Email/Password

Write the code below

828f26b

Send

# Online results submission

- many numerical values

i	z1	z2	z3	z4	z5	z6	z7
	148.33	155.88	202.12	164.35	180.91	30.29	185.19
	25.97	153.5	34.64	35.79	55.56	26.212	10,693
	0	0	0	0	0	0	0
	50	50	50	50	50	50	50



# Online results submission

Grade = Quality of the work +  
+ Quality of the submission

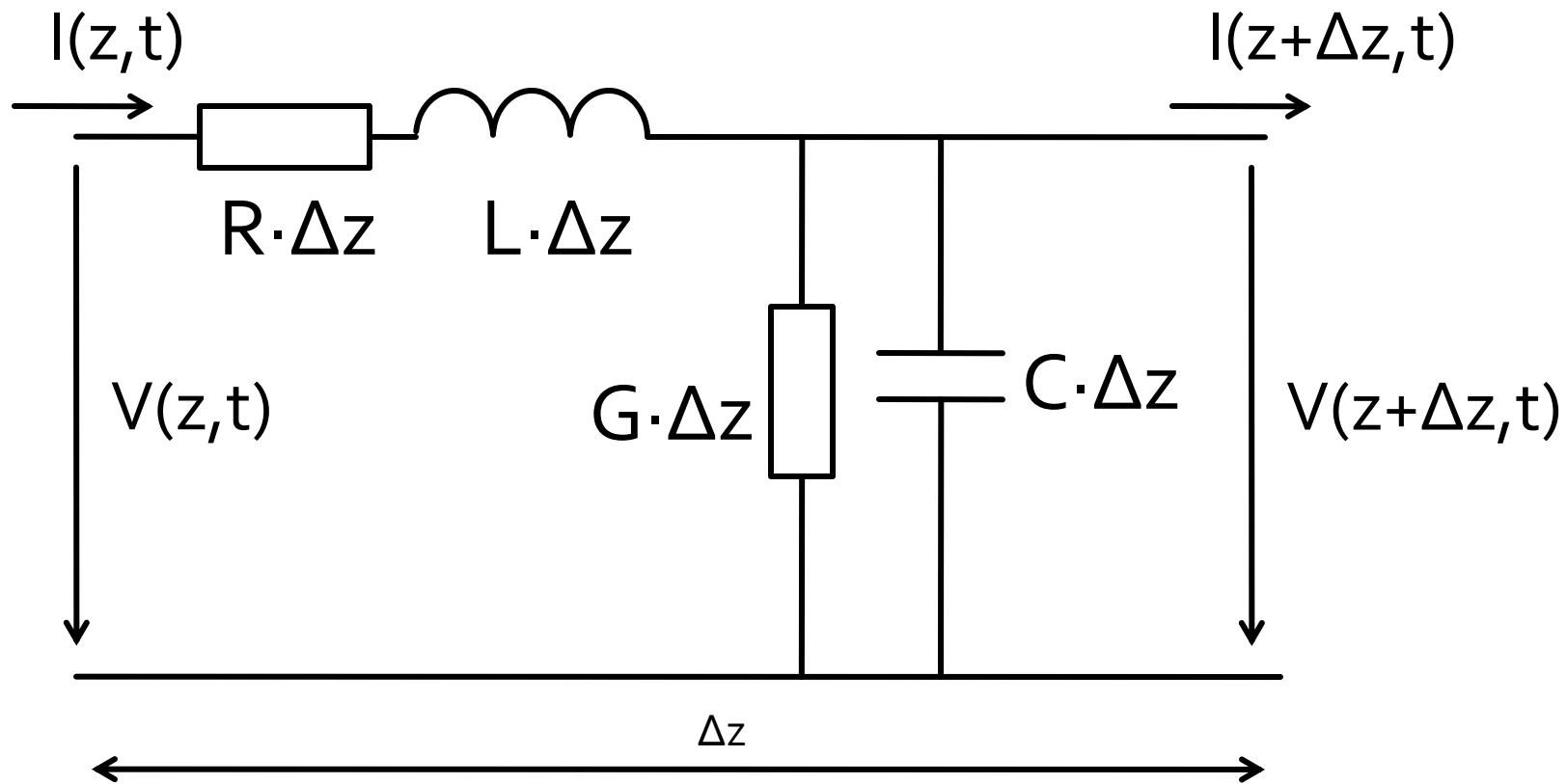
# TEM transmission lines

# Course Topics

- **Transmission lines**
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers?~~

# Transmission line equivalent model

- TEM wave propagation, at least two conductors



# Telegrapher's equations

- time domain

$$\frac{\partial v(z,t)}{\partial z} = -R \cdot i(z,t) - L \cdot \frac{\partial i(z,t)}{\partial t} \quad K \parallel$$

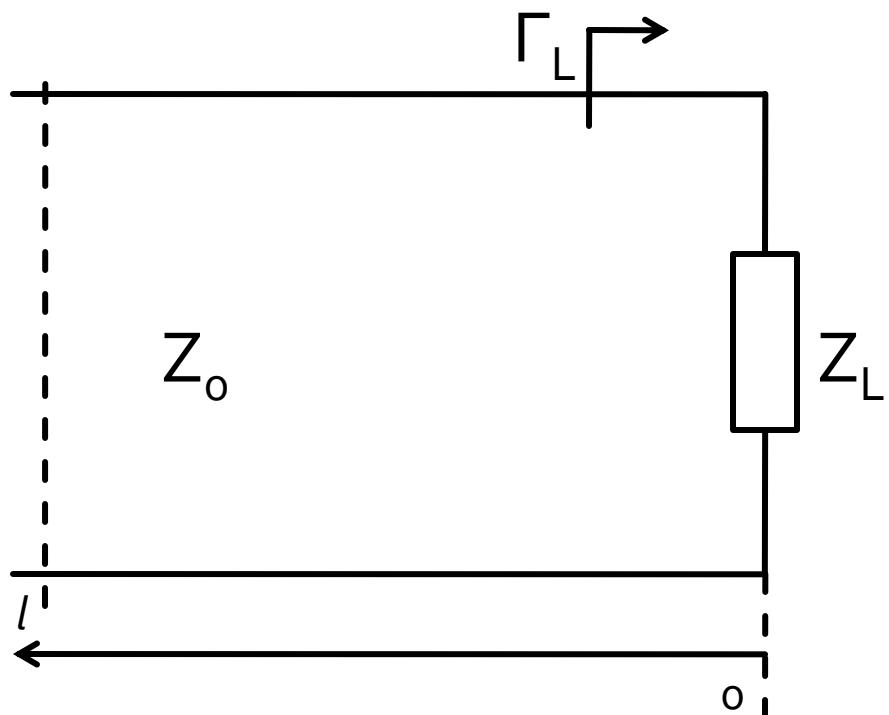
$$\frac{\partial i(z,t)}{\partial z} = -G \cdot v(z,t) - C \cdot \frac{\partial v(z,t)}{\partial t} \quad K \perp$$

- harmonic signals (frequency domain)

$$\frac{dV(z)}{dz} = -(R + j \cdot \omega \cdot L) \cdot I(z) \quad \left/ \frac{d}{dz} (\dots) \right.$$

$$\frac{dI(z)}{dz} = -(G + j \cdot \omega \cdot C) \cdot V(z)$$

# The lossless line



$$V(z) = V_0^+ e^{-j\beta z} + V_0^- e^{j\beta z}$$

$$I(z) = \frac{V_0^+}{Z_0} e^{-j\beta z} - \frac{V_0^-}{Z_0} e^{j\beta z}$$

$$Z_L = \frac{V(0)}{I(0)} \quad Z_L = \frac{V_0^+ + V_0^-}{V_0^+ - V_0^-} \cdot Z_0$$

- voltage reflection coefficient

$$\boxed{\Gamma = \frac{V_0^-}{V_0^+} = \frac{Z_L - Z_0}{Z_L + Z_0}}$$

- $Z_0$  real

# The lossless line

$$V(z) = V_0^+ \cdot (e^{-j\beta z} + \Gamma \cdot e^{j\beta z}) \quad I(z) = \frac{V_0^+}{Z_0} \cdot (e^{-j\beta z} - \Gamma \cdot e^{j\beta z})$$

- time-average Power flow along the line

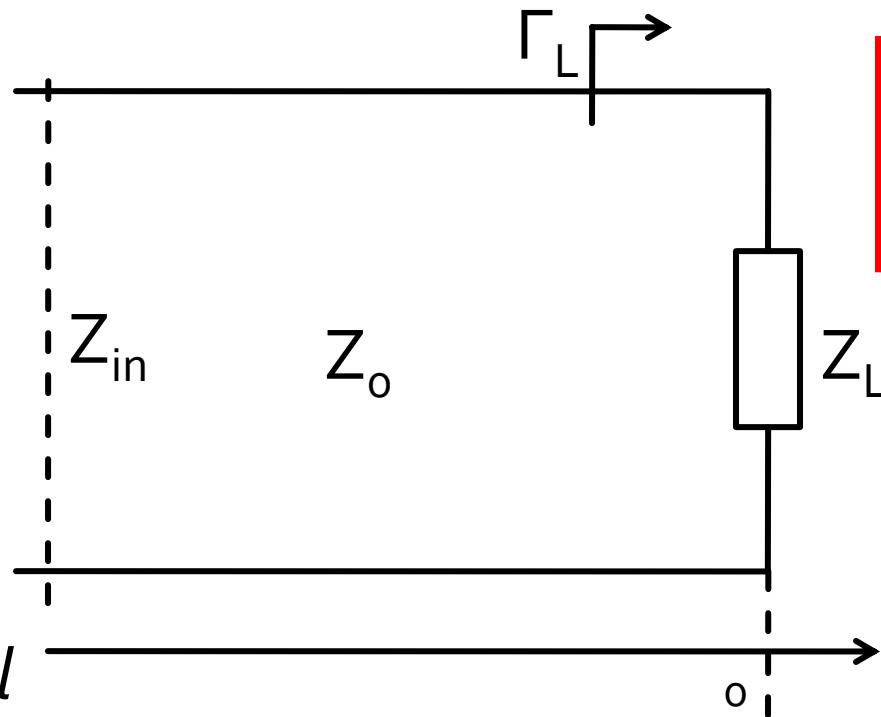
$$P_{avg} = \frac{1}{2} \cdot \text{Re}\{V(z) \cdot I(z)^*\} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \text{Re}\left\{1 - \Gamma^* \cdot e^{-2j\beta z} + \Gamma \cdot e^{2j\beta z} - |\Gamma|^2\right\}$$
$$P_{avg} = \frac{1}{2} \cdot \frac{|V_0^+|^2}{Z_0} \cdot \left(1 - |\Gamma|^2\right)$$

$(z - z^*) = \text{Im}$

- Total power delivered to the load = Incident power – “Reflected” power
- Return “Loss” [dB] 
$$\text{RL} = -20 \cdot \log|\Gamma| \quad [\text{dB}]$$

# The lossless line

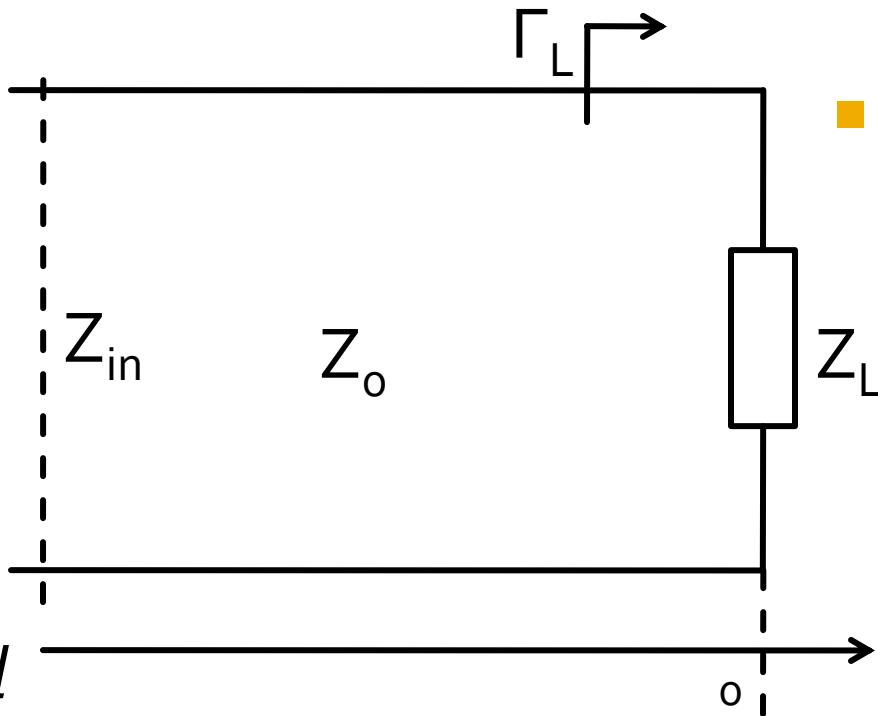
- input impedance of a length  $l$  of transmission line with characteristic impedance  $Z_0$ , loaded with an arbitrary impedance  $Z_L$



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

# The lossless line, special cases

- $l = k \cdot \lambda / 2$        $\beta \cdot l = \frac{2\pi}{\lambda} \cdot l = k \cdot \pi$        $\tan \beta \cdot l = 0$        $Z_{in} = Z_0$
- $l = \lambda / 4 + k \cdot \lambda / 2$        $\tan \beta \cdot l \rightarrow \infty$        $Z_{in} = \frac{Z_0^2}{Z_L}$



■ quarter-wave transformer

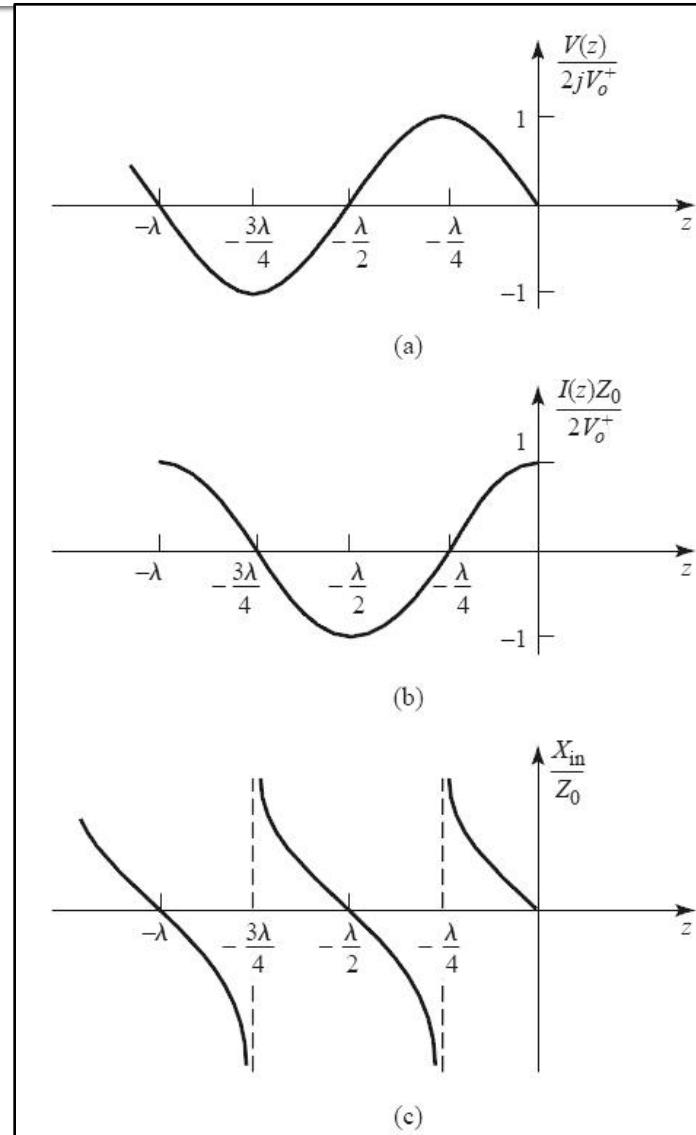
$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

# Short-circuited transmission line

- $Z_L = 0$
- purely imaginary for any length  $l$ 
  - $\pm \rightarrow$  depending on  $l$  value

$$Z_{in} = j \cdot Z_0 \cdot \tan \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

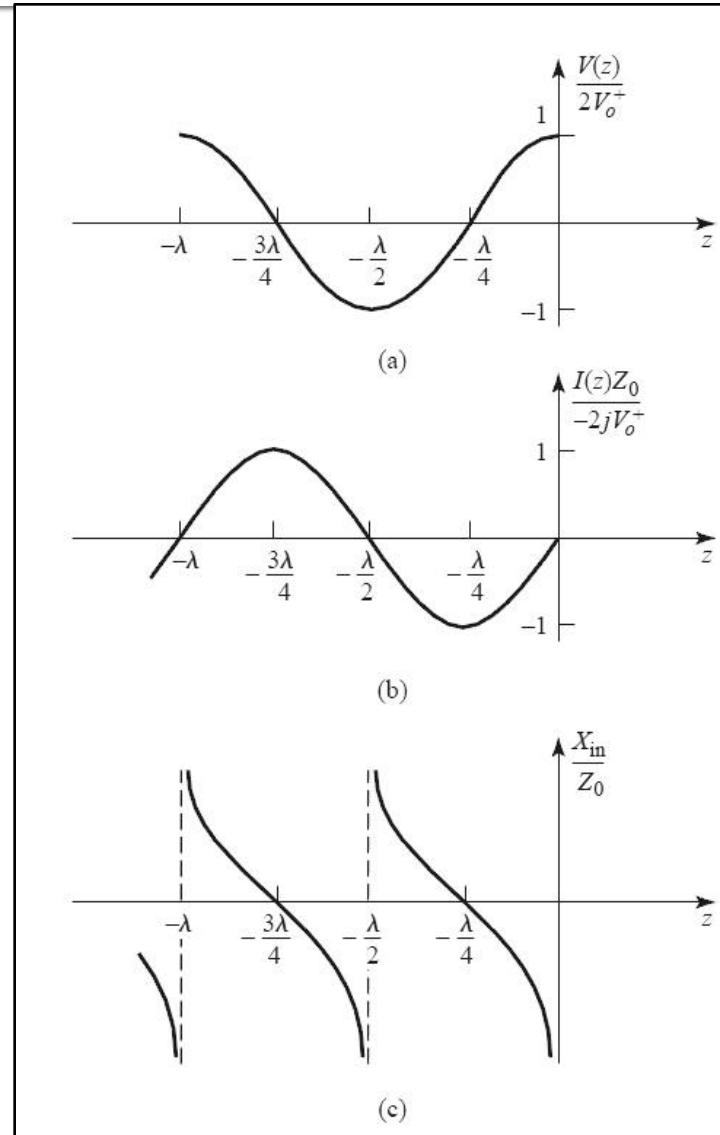


# Open-circuited transmission line

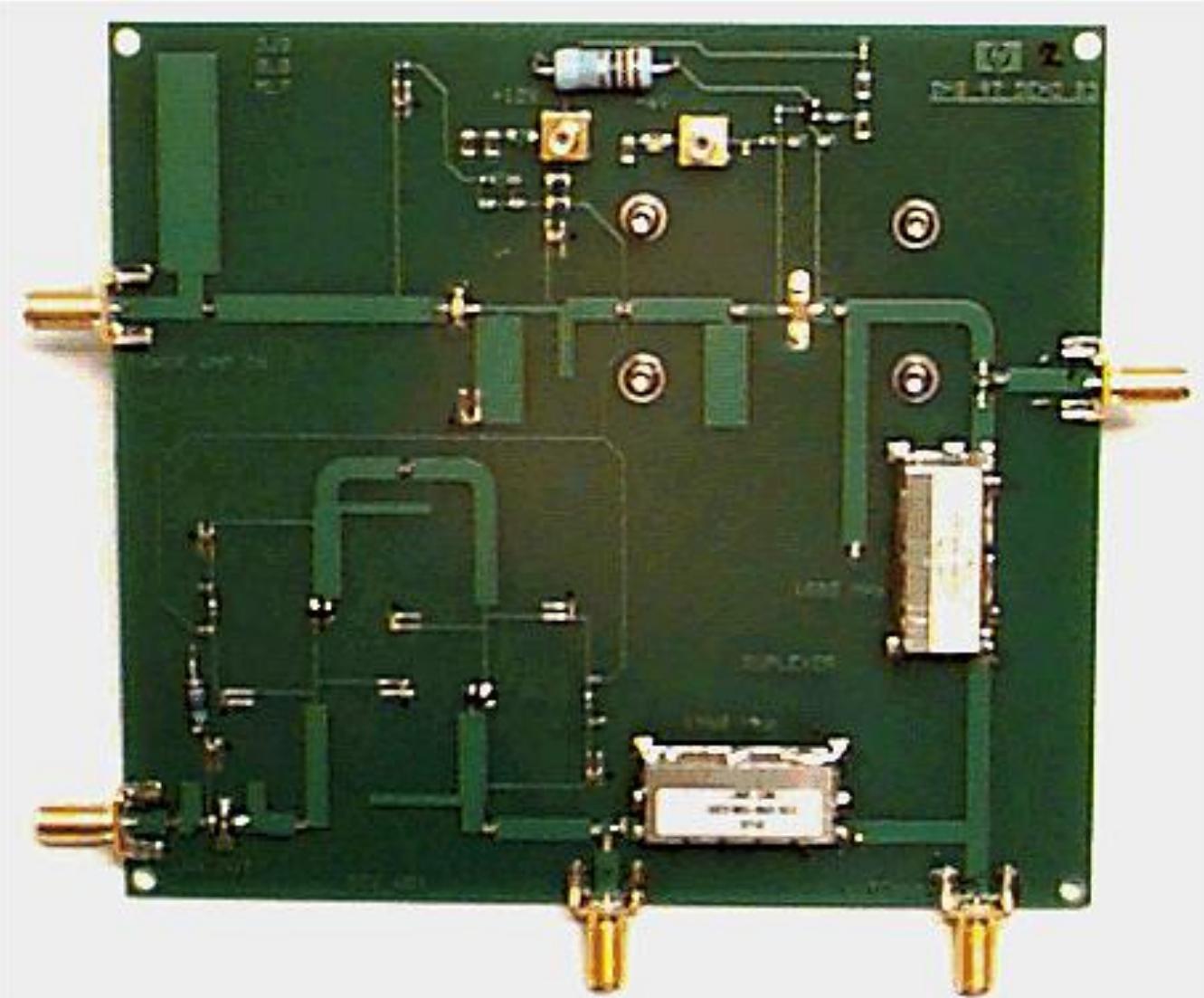
- $Z_L = \infty \rightarrow 1/Z_L = 0$
- purely imaginary for any length  $l$ 
  - $\pm/- \rightarrow$  depending on  $l$  value

$$Z_{in} = -j \cdot Z_0 \cdot \cot \beta \cdot l$$

$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan(\beta \cdot l)}{Z_0 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$



# Examples



# Voltage standing wave ratio

$$V(z) = V_0^+ \cdot (e^{-j\beta z} + \Gamma \cdot e^{j\beta z}) \quad |V(z)| = |V_0^+| \cdot |e^{-j\beta z}| \cdot |1 + \Gamma \cdot e^{2j\beta z}| \quad \Gamma = |\Gamma| \cdot e^{j\theta}$$

$$|V(z)| = |V_0^+| \cdot |1 + |\Gamma| \cdot e^{\theta + 2j\beta z}|$$

maximum magnitude value for  $e^{\theta + 2j\beta z} = 1$

$$V_{\max} = |V_0^+| \cdot (1 + |\Gamma|)$$

minimum magnitude value for  $e^{\theta + 2j\beta z} = -1$

$$V_{\min} = |V_0^+| \cdot (1 - |\Gamma|)$$

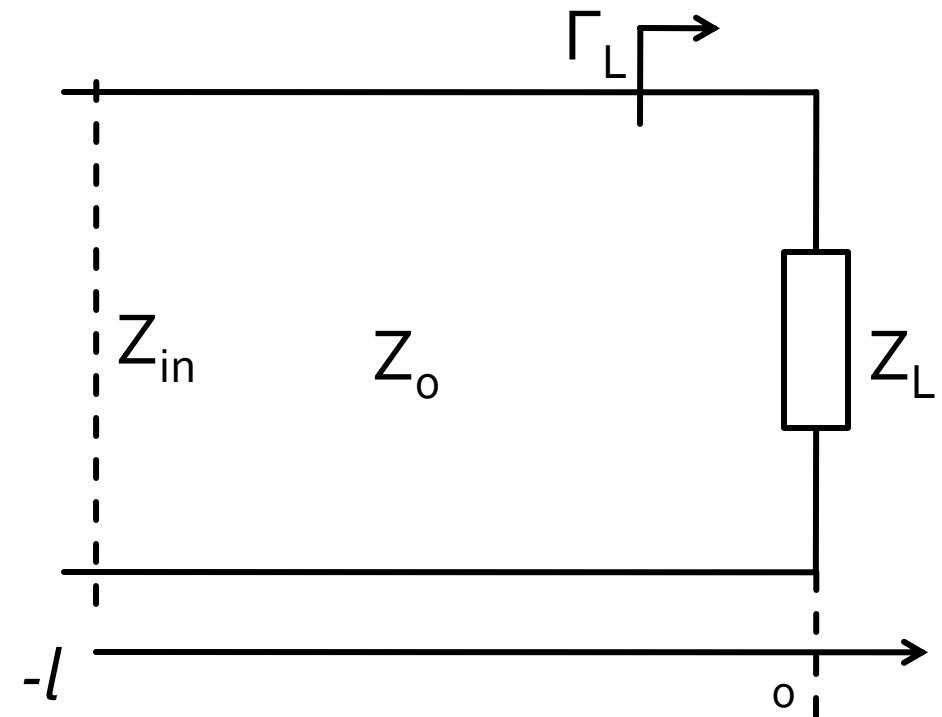
- SWR is defined as the ratio between maximum and minimum

- (Voltage) Standing Wave Ratio

$$VSWR = \frac{V_{\max}}{V_{\min}} = \frac{1 + |\Gamma|}{1 - |\Gamma|}$$

- real number  $1 \leq VSWR < \infty$
  - a measure of the mismatch (SWR = 1 means a matched line)

# The lossless line +/-



$$Z_{in} = Z_0 \cdot \frac{Z_L + j \cdot Z_0 \cdot \tan \beta \cdot l}{Z_0 + j \cdot Z_L \cdot \tan \beta \cdot l}$$

$$V(z) = V_0^+ e^{-\gamma \cdot z} + V_0^- e^{\gamma \cdot z}$$

$$I(z) = I_0^+ e^{-\gamma \cdot z} + I_0^- e^{\gamma \cdot z}$$

$$\Gamma(-l) = \Gamma(0) \cdot e^{-2j\beta l}$$

$$\Gamma_{in} = \Gamma_L \cdot e^{-2j\beta l}$$

Power transfer

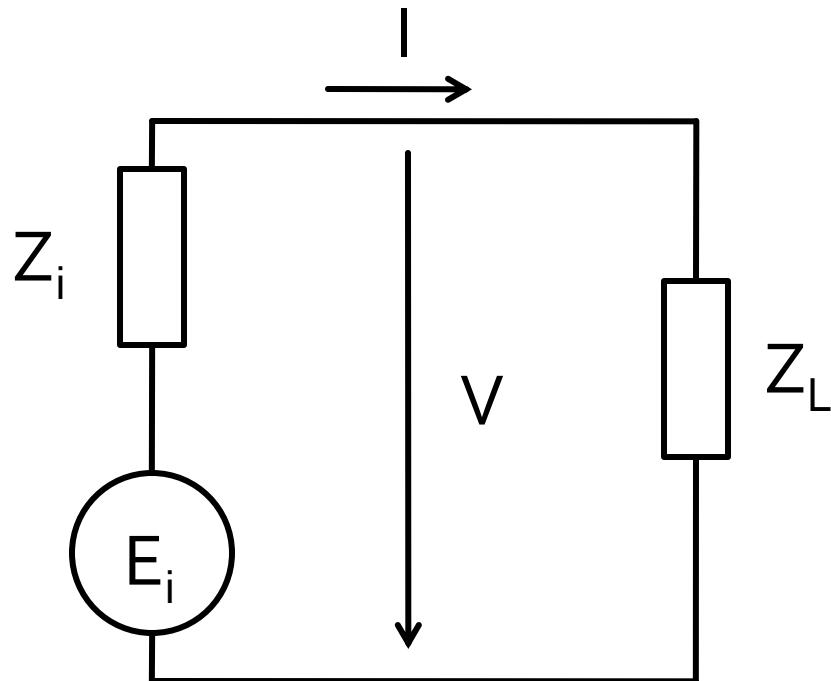
# Impedance Matching

# Course Topics

- Transmission lines
- Impedance matching and tuning
- Directional couplers
- Power dividers
- Microwave amplifier design
- Microwave filters
- ~~Oscillators and mixers?~~

# Matching

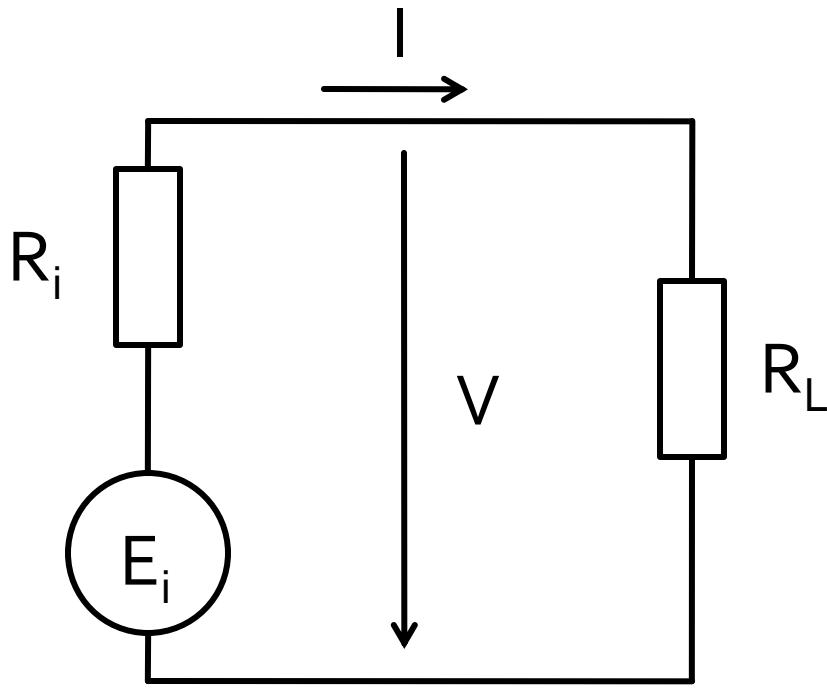
- Source matched to load ?



- impedance values ?
- existence of reflections ?

# Matching, real impedances

- Source matched to load



$$I = \frac{E_i}{R_i + R_L}$$

$$V = \frac{E_i \cdot R_L}{R_i + R_L}$$

$$P_L = R_L \cdot I^2$$

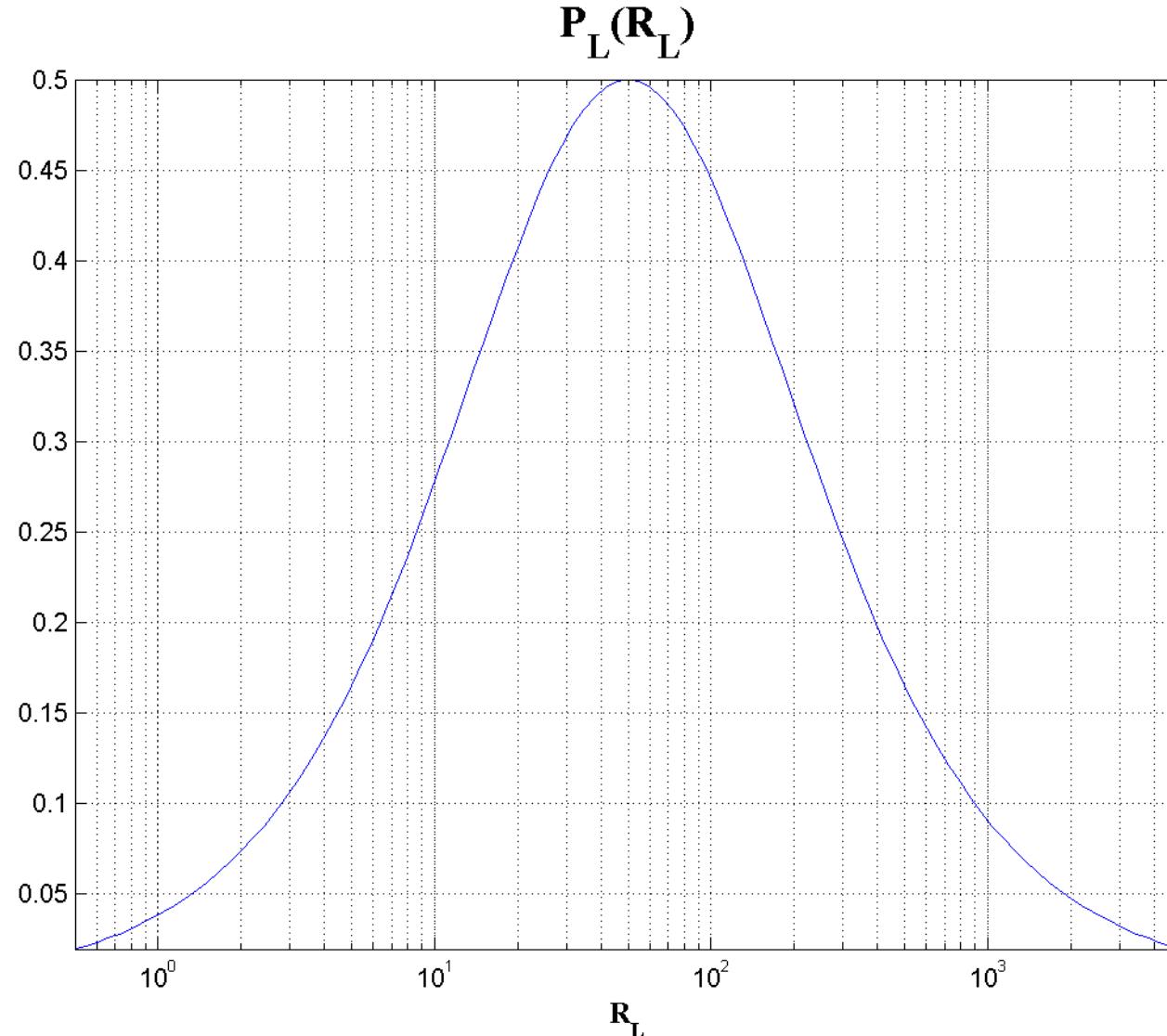
$$P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

# Matching, real impedances

$$P_L = R_L \cdot I^2 \quad P_L = \frac{R_L \cdot E_i^2}{(R_i + R_L)^2}$$

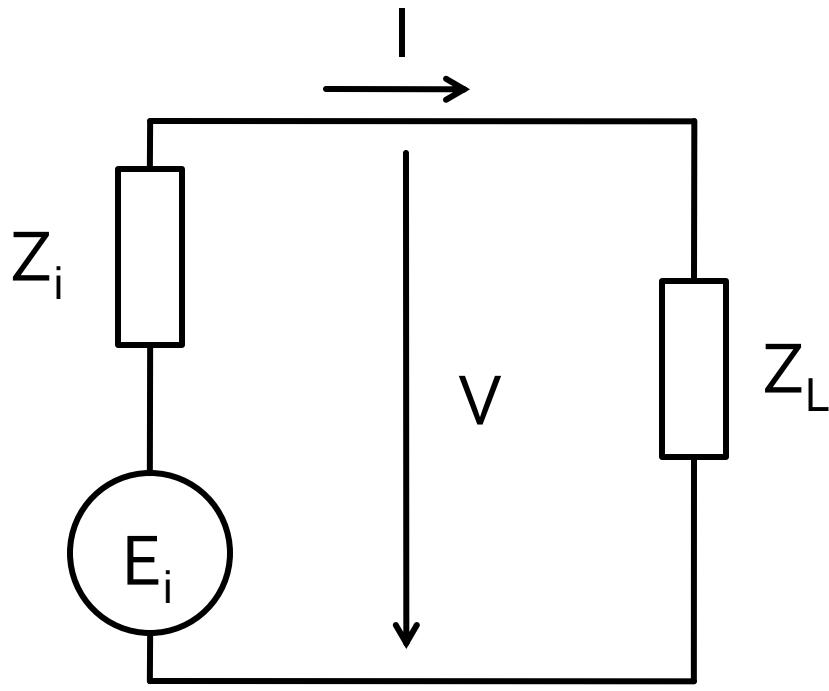
- Power dissipated on load
  - $R_i = 50\Omega$
  - $R_L = 0 \rightarrow P_L = 0$
  - $R_L = \infty \rightarrow P_L = 0$

# Matching, real impedances



# Matching, complex impedances

- Source matched to load



$$I = \frac{E_i}{Z_i + Z_L}$$

$$V = \frac{E_i \cdot Z_L}{Z_i + Z_L}$$

$$P_L = \text{Re}\left\{ Z_L \cdot |I|^2 \right\}$$

$$P_L = \text{Re}\left\{ Z_L \right\} \cdot \left| \frac{E_i}{Z_i + Z_L} \right|^2$$

# Matching

$$P_L = \frac{R_L \cdot |E_i|^2}{|Z_i + Z_L|^2} = \frac{R_L \cdot |E_i|^2}{|(R_i + R_L) + j \cdot (X_i + X_L)|^2}$$

$$|a + j \cdot b| = \sqrt{a^2 + b^2}$$

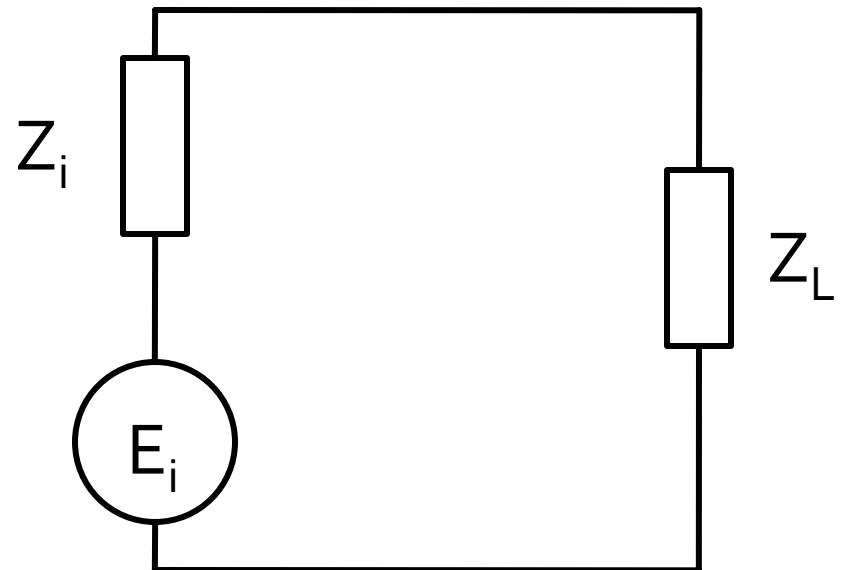
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

- Matching
  - maximum power transmitted to the load
  - condition?

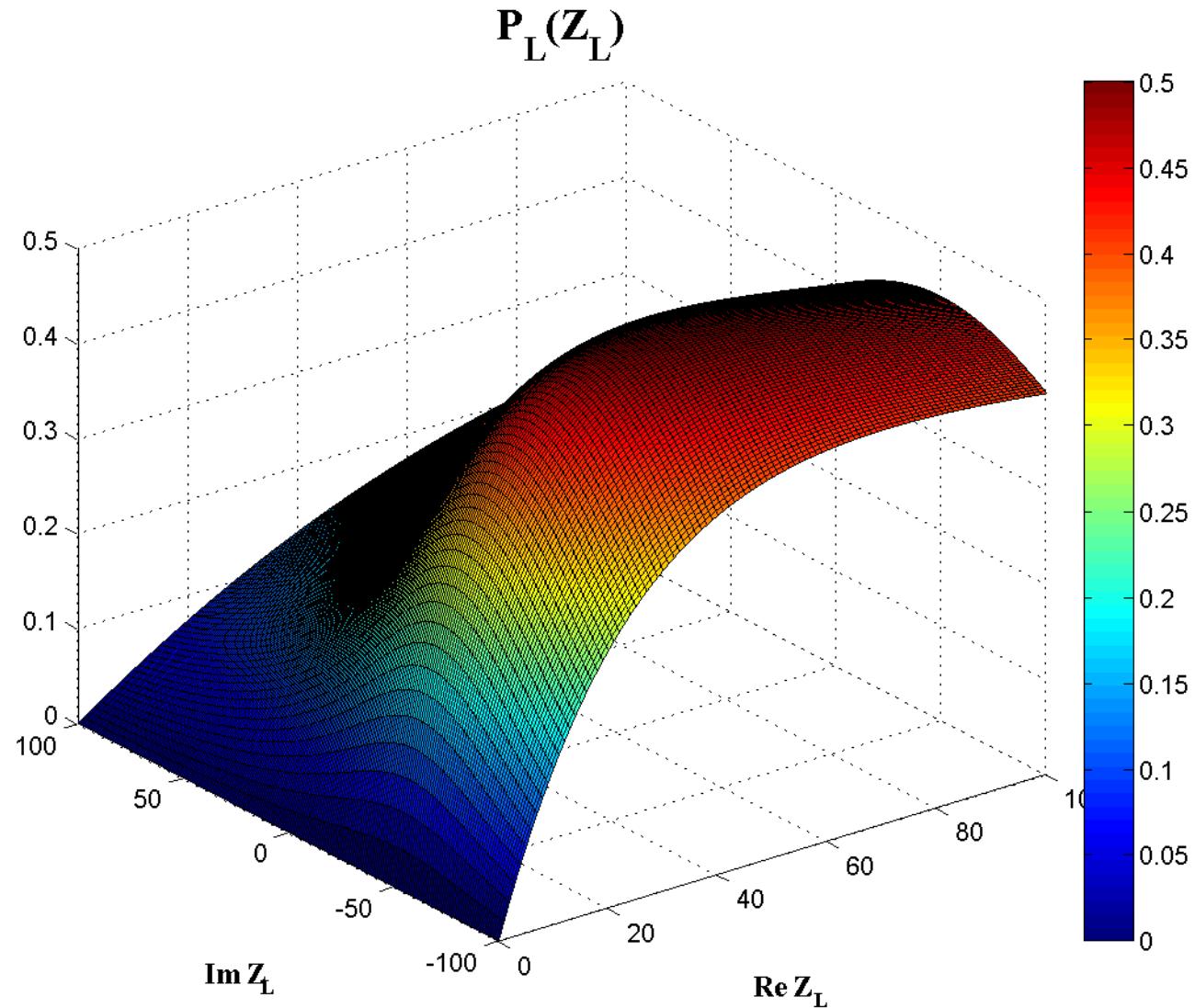
# Matching, example

- $E = 10V$
- $Z_i = 50 \Omega + j \cdot 50\Omega$
- $P_L(Z_L)$  ?

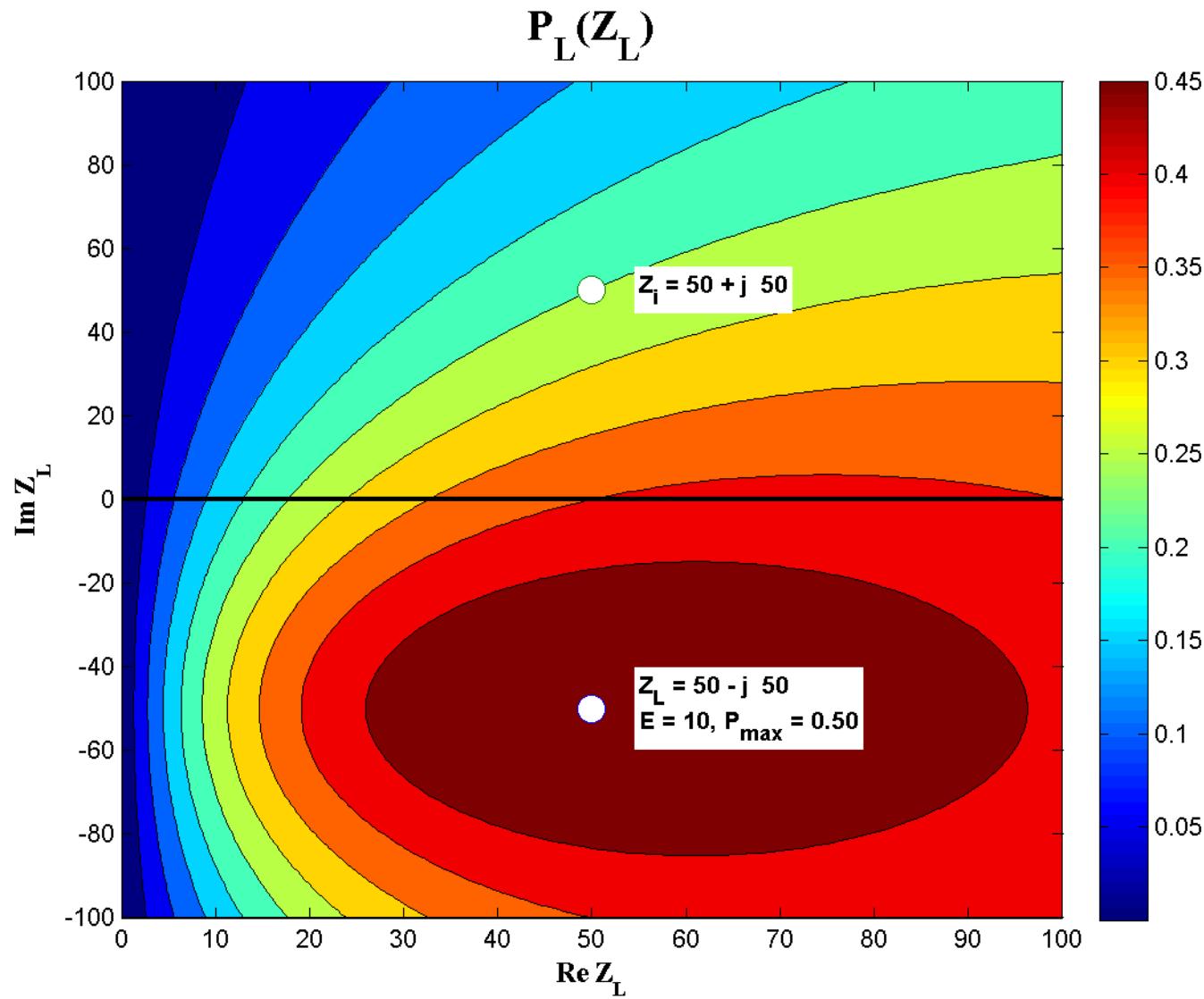
$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$



# Matching, example



# Matching, example



# Matching , from the point of view of power transmission

$$R_i > 0, R_L > 0$$

$$P_L = \frac{|E_i|^2}{4R_i + \frac{(R_i - R_L)^2}{R_L} + \frac{(X_i + X_L)^2}{R_L}}$$

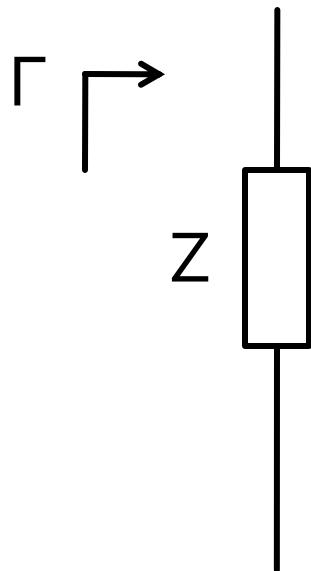
$$P_{L\max} = \frac{|E_i|^2}{4R_i} \equiv P_a \quad R_L = R_i, X_L = -X_i$$

- $P_a$  : Available Power

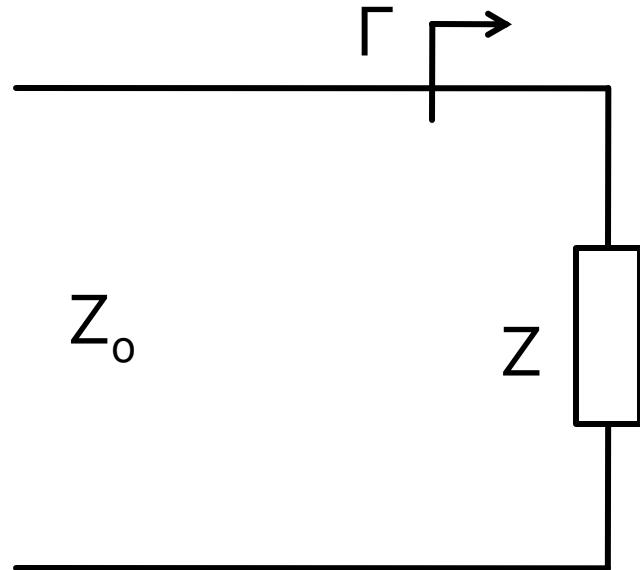
$$Z_L = Z_i^*$$

# Reflection coefficient

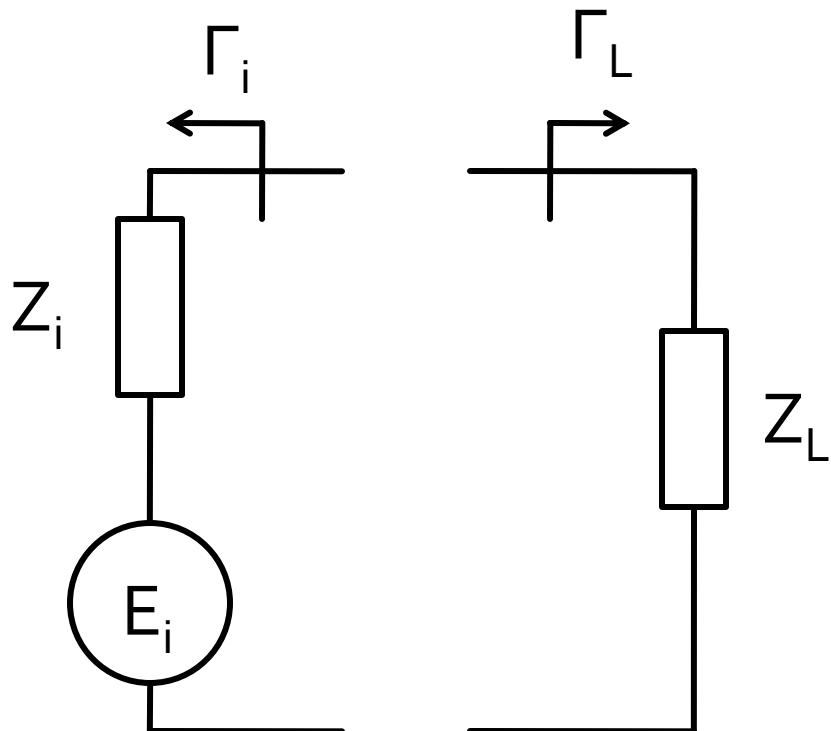
- Any impedance  $Z_0$  chosen as reference



$$\Gamma = \frac{Z - Z_0^*}{Z + Z_0}$$



# Matching , from the point of view of power transmission



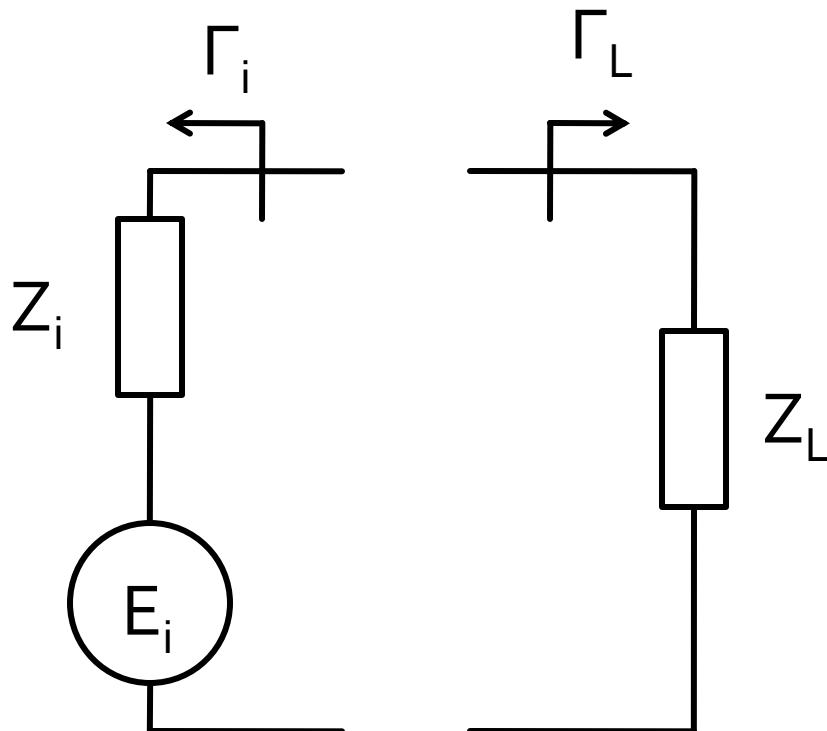
$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0}$$

$$\Gamma_i = \frac{(R_i - R_0) + j \cdot (X_i + X_0)}{(R_i + R_0) + j \cdot (X_i + X_0)}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0}$$

$$\Gamma_L = \frac{(R_L - R_0) + j \cdot (X_L + X_0)}{(R_L + R_0) + j \cdot (X_L + X_0)}$$

# Matching , from the point of view of power transmission



$$\Gamma_i = \frac{Z_i - Z_0^*}{Z_i + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_i + Z_0}$$

$$\Gamma_L = \frac{Z_L - Z_0^*}{Z_L + Z_0} = 1 - \frac{Z_0 + Z_0^*}{Z_L + Z_0}$$

$$\Gamma_i^* = 1 - \frac{Z_0^* + Z_0}{Z_i^* + Z_0} = 1 - \frac{Z_0^* + Z_0}{Z_L + Z_0^*}$$

# Matching , from the point of view of power transmission

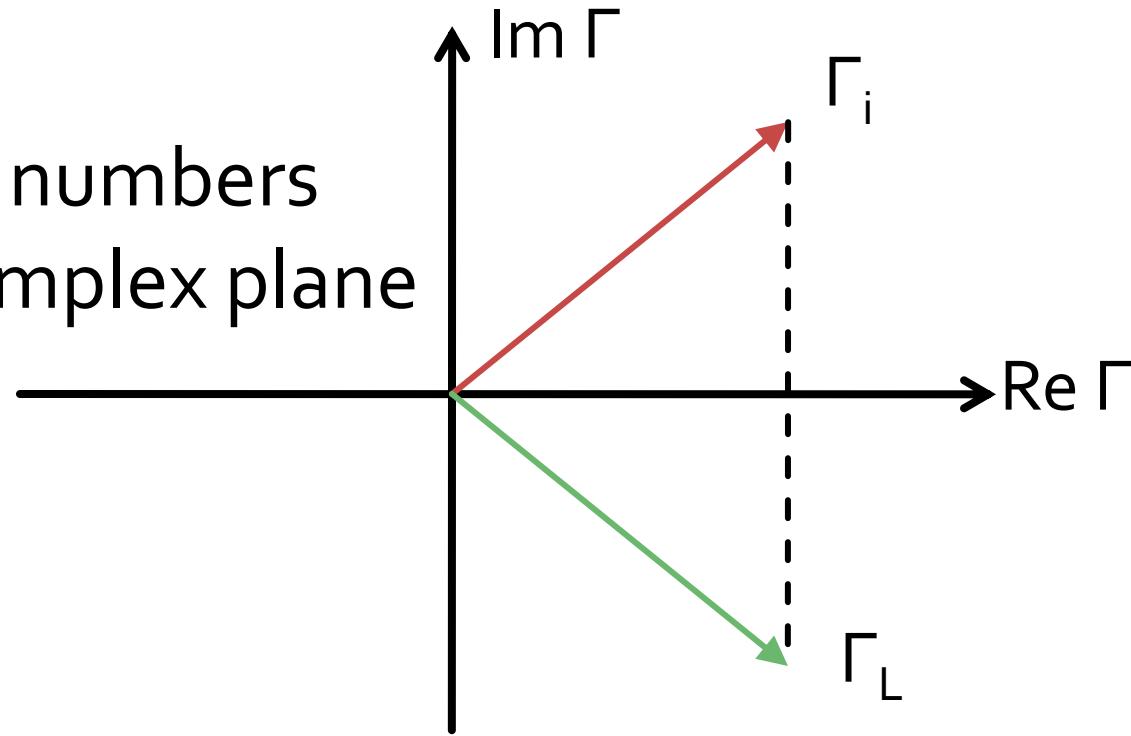
$$Z_L = Z_i^*$$

If we choose a (any) real  $Z_0$

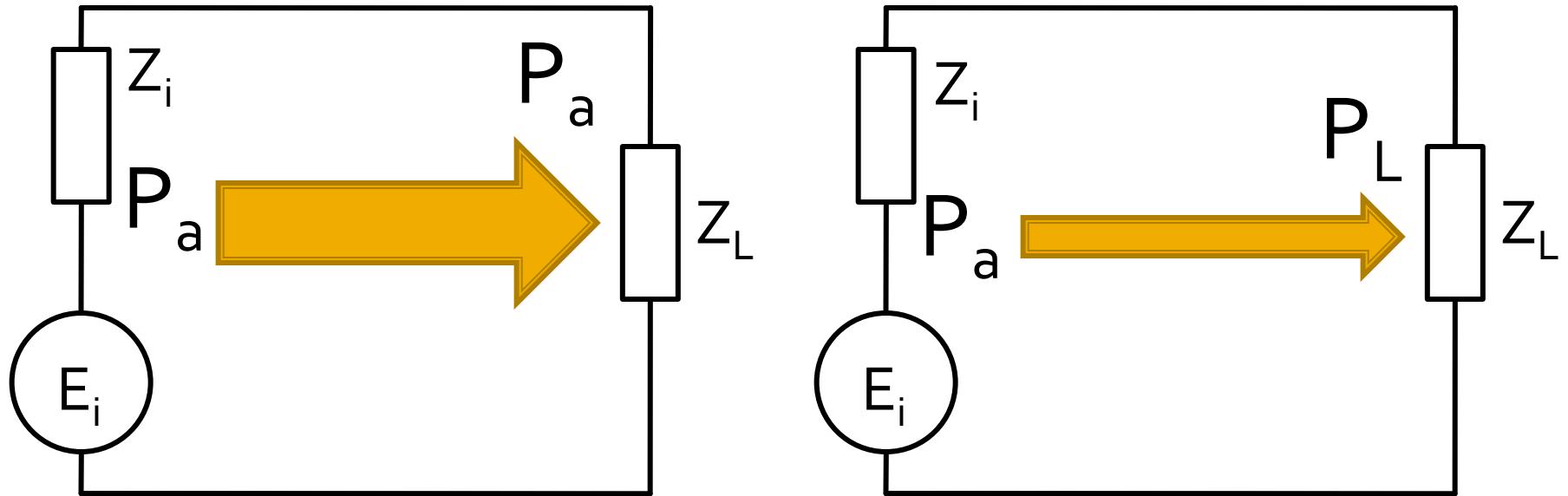
$$\Gamma = \frac{Z - Z_0}{Z + Z_0}$$

$$\Gamma_L = \Gamma_i^*$$

- complex numbers
- in the complex plane



# Reflection and power / Model



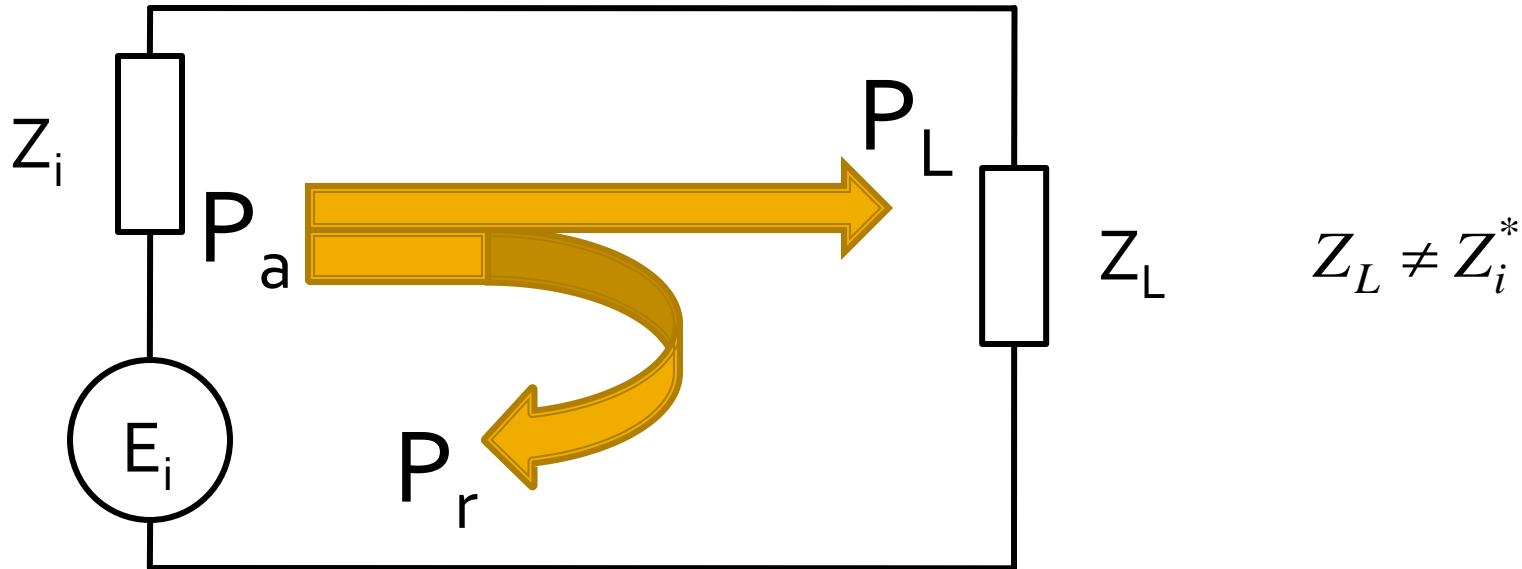
$$Z_L = Z_i^*$$

$$P_L = P_a$$

$$Z_L \neq Z_i^*$$

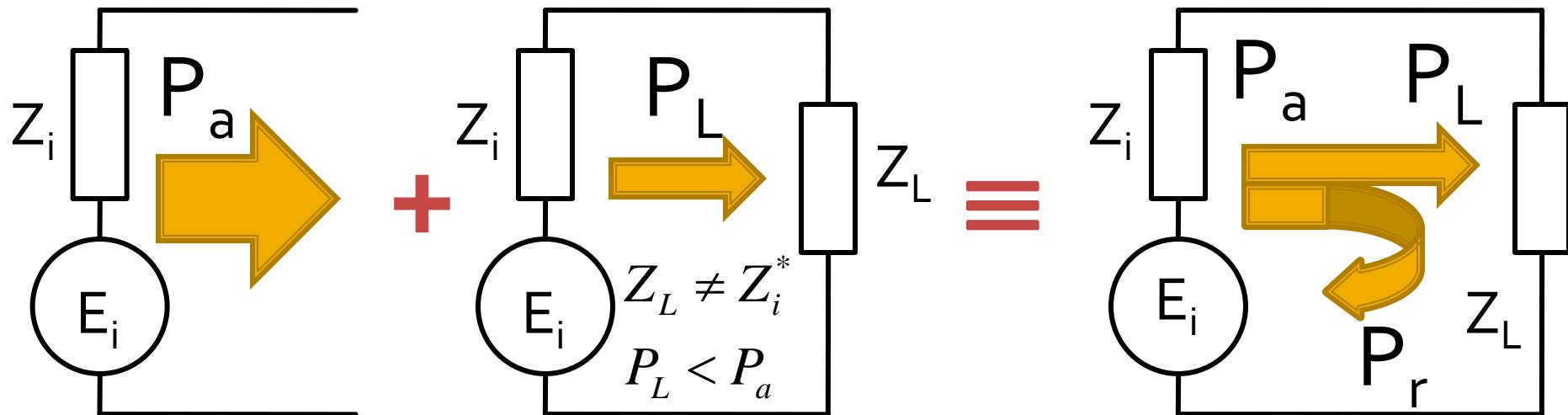
$$P_L < P_a$$

# Reflection and power / Model



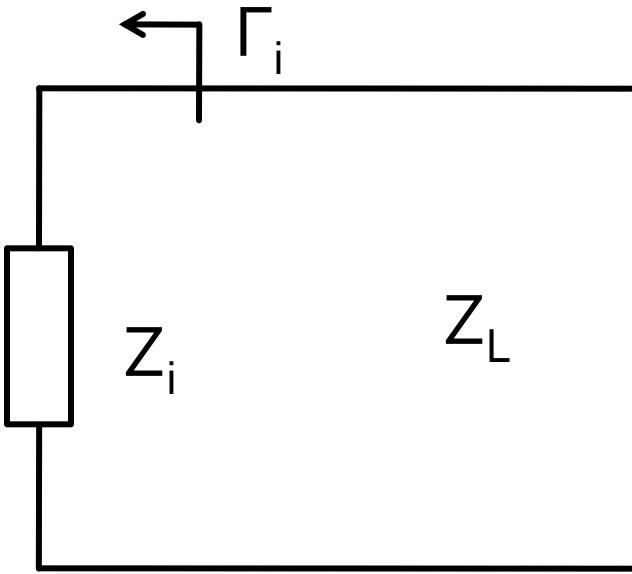
- Power reflection
- Power of the reflected wave

# Reflection and power / Model

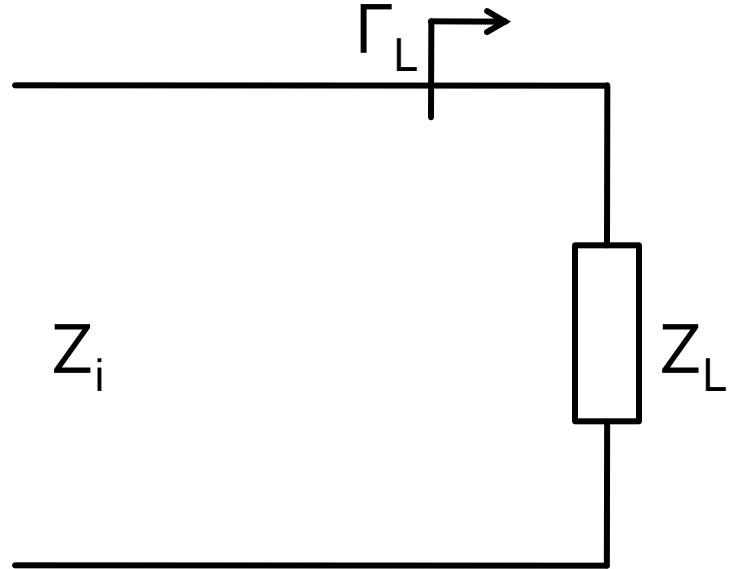


- The source has the ability to send to the load a certain maximum power (available power)  $P_a$
- For a particular load the power sent to the load is less than the maximum (mismatch)  $P_L < P_a$
- The phenomenon is “as if” (model) some of the power is reflected  $P_r = P_a - P_L$
- The power is a **scalar** !

# Reflection coefficient



$$\Gamma_i = \frac{Z_i - Z_L^*}{Z_i + Z_L}$$



$$\Gamma_L = \frac{Z_L - Z_i^*}{Z_L + Z_i}$$

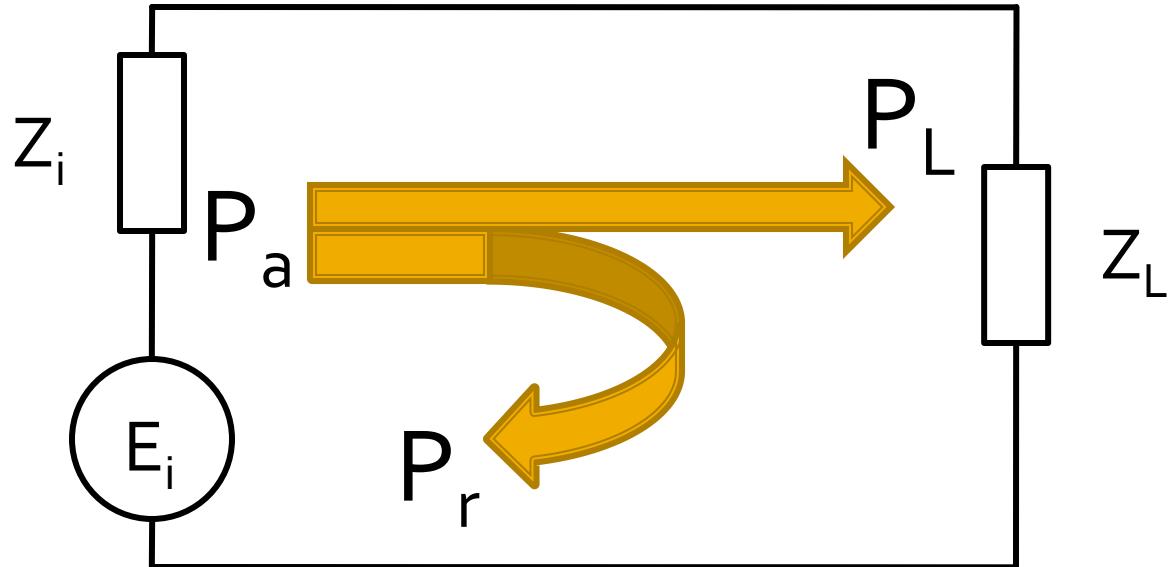
# Reflection coefficient

$$\Gamma_i = \frac{(R_i - R_L) + j \cdot (X_i + X_L)}{(R_i + R_L) + j \cdot (X_i + X_L)} \quad \Gamma_L = \frac{(R_L - R_i) + j \cdot (X_L + X_i)}{(R_L + R_i) + j \cdot (X_L + X_i)}$$

$$|\Gamma_i| = \frac{|(R_i - R_L) + j \cdot (X_i + X_L)|}{|(R_i + R_L) + j \cdot (X_i + X_L)|} = \frac{\sqrt{(R_i - R_L)^2 + (X_i + X_L)^2}}{\sqrt{(R_i + R_L)^2 + (X_i + X_L)^2}} = |\Gamma_L|$$

$$|\Gamma_i| = |\Gamma_L| \equiv |\Gamma|$$

# Reflection and power / Model



$$P_a = \frac{|E_i|^2}{4R_i}$$

$$P_L = \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2}$$

$$P_r = P_a - P_L = \frac{|E_i|^2}{4R_i} - \frac{R_L \cdot |E_i|^2}{(R_i + R_L)^2 + (X_i + X_L)^2} = \frac{|E_i|^2}{4R_i} \cdot \left[ 1 - \frac{4R_L \cdot R_i}{(R_i + R_L)^2 + (X_i + X_L)^2} \right]$$

$$P_r = \frac{|E_i|^2}{4R_i} \cdot \left[ \frac{(R_i - R_L)^2 + (X_i + X_L)^2}{(R_i + R_L)^2 + (X_i + X_L)^2} \right] = P_a \cdot |\Gamma|^2$$

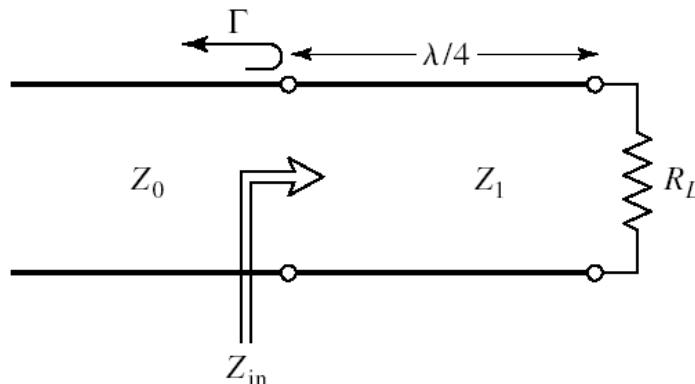
- $|\Gamma|^2$  is a power reflection coefficient

The quarter-wave transformer

# Impedance Matching

# The quarter-wave transformer

- Feed line – input line with characteristic impedance  $Z_o$
- **Real** load impedance  $R_L$
- We desire matching the load to the fider with a second line with the length  $\lambda/4$  and characteristic impedance  $Z_1$

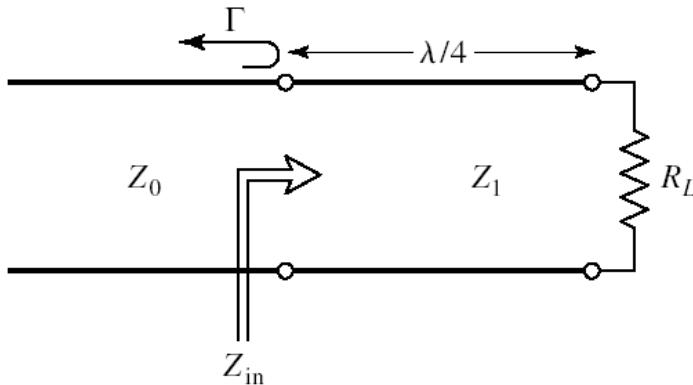


$$Z_{in} = Z_1 \frac{1 + \Gamma e^{-2j\beta l}}{1 - \Gamma e^{-2j\beta l}}$$

$$\Gamma_o = \frac{V_0^-}{V_0^+} = \frac{R_L - Z_1}{R_L + Z_1}$$

$$Z_{in} = Z_1 \frac{R_L + jZ_1 \tan(\beta l)}{Z_1 + jR_L \tan(\beta l)}$$

# The quarter-wave transformer



$$\Gamma_{in} = \frac{Z_{in} - Z_0}{Z_{in} + Z_0}$$

$$\beta \cdot l = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$$

$$Z_{in} = \frac{Z_1^2}{R_L}$$

$$\Gamma_{in} = \frac{Z_1^2 - Z_0 \cdot R_L}{Z_1^2 + Z_0 \cdot R_L} \quad \Gamma_{in} = 0 \quad Z_1 = \sqrt{Z_0 R_L}$$

- In the feed line ( $Z_0$ ) we have only progressive wave
- In the quarter-wave line ( $Z_1$ ) we have standing waves

# The quarter-wave transformer

- The Multiple-Reflection Viewpoint

$$\Gamma = \Gamma_1 - T_1 T_2 \Gamma_3 + T_1 T_2 \Gamma_2 \Gamma_3^2 - T_1 T_2 \Gamma_2^2 \Gamma_3^3 + \dots$$

$$= \Gamma_1 - T_1 T_2 \Gamma_3 \sum_{n=0}^{\infty} (-\Gamma_2 \Gamma_3)^n.$$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0},$$

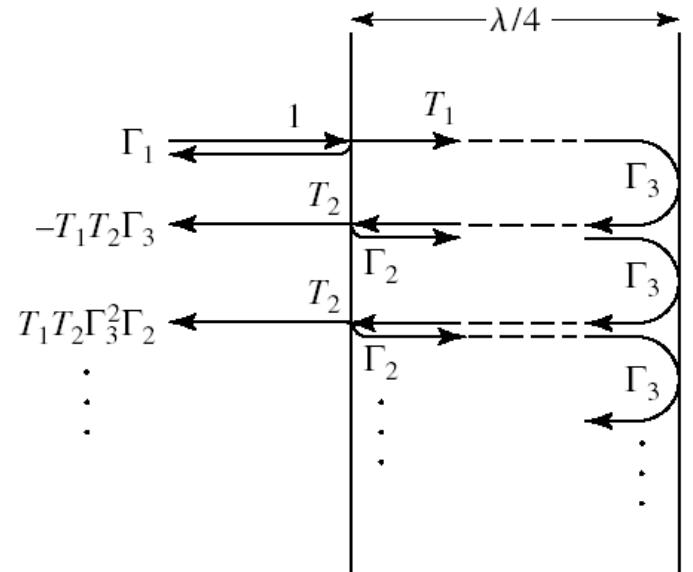
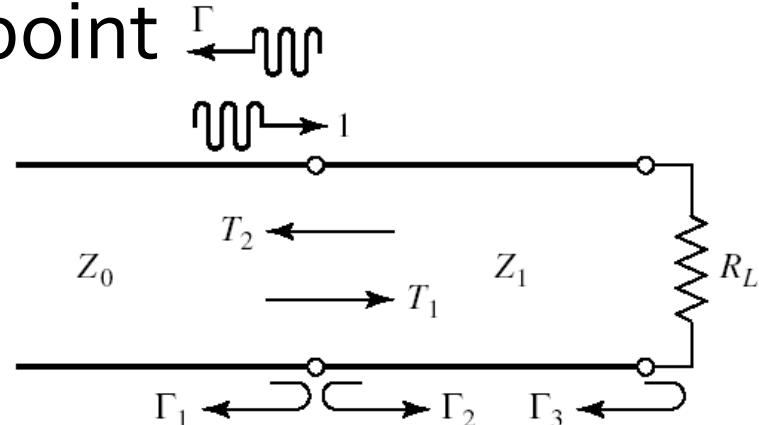
$$\Gamma_2 = \frac{Z_0 - Z_1}{Z_0 + Z_1} = -\Gamma_1,$$

$$\Gamma_3 = \frac{R_L - Z_1}{R_L + Z_1},$$

$$T_1 = \frac{2Z_1}{Z_1 + Z_0},$$

$$T_2 = \frac{2Z_0}{Z_1 + Z_0}.$$

$$T = 1 - \Gamma$$



# The quarter-wave transformer

- The Multiple-Reflection Viewpoint

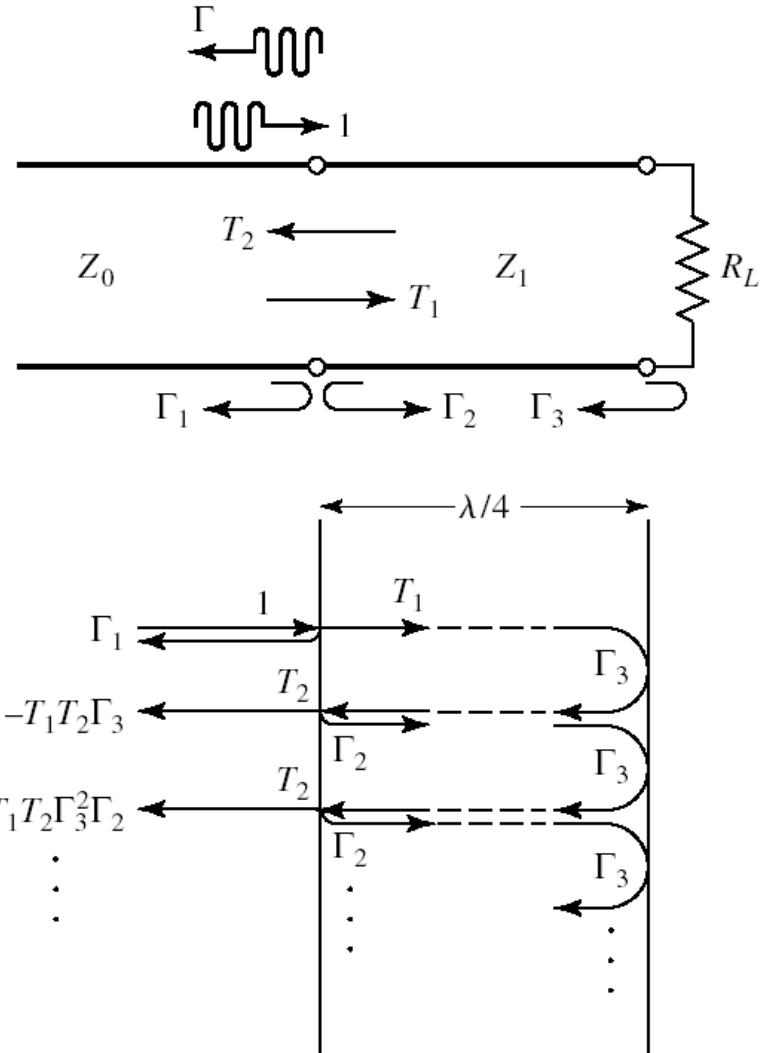
$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}, \quad \text{for } |x| < 1,$$

$$\Gamma = \Gamma_1 - \frac{T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3} = \frac{\Gamma_1 + \Gamma_1 \Gamma_2 \Gamma_3 - T_1 T_2 \Gamma_3}{1 + \Gamma_2 \Gamma_3}.$$

$$\Gamma_1 - \Gamma_3 (\Gamma_1^2 + T_1 T_2) = \frac{2(Z_1^2 - Z_0 R_L)}{(Z_1 + Z_0)(R_L + Z_1)},$$

$$\Gamma_1^2 + T_1 T_2 = \frac{(Z_1 - Z_0)^2}{(Z_1 + Z_0)^2} + \frac{4Z_1 Z_0}{(Z_1 + Z_0)^2} = 1$$

$$\Gamma = 0 \leftrightarrow Z_1^2 - Z_0 \cdot R_L = 0$$



# Frequency response

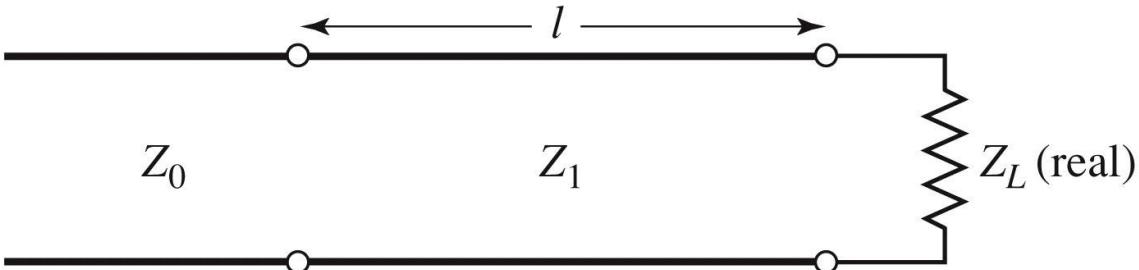


Figure 5.10  
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$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot \tan(\beta \cdot l)}{Z_1 + j \cdot Z_L \cdot \tan(\beta \cdot l)}$$

$$Z_{in} = Z_1 \cdot \frac{Z_L + j \cdot Z_1 \cdot t}{Z_1 + j \cdot Z_L \cdot t}$$

$$\Gamma = \frac{Z_{in} - Z_0}{Z_{in} + Z_0} = \frac{Z_1(Z_L - Z_0) + jt(Z_1^2 - Z_0 Z_L)}{Z_1(Z_L + Z_0) + jt(Z_1^2 + Z_0 Z_L)}.$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0 + j2t\sqrt{Z_0 Z_L}}.$$

$$Z_1 = \sqrt{Z_0 \cdot Z_L}$$

- (only) at  $f_o$

$$l = \frac{\lambda_0}{4} \quad \beta_0 \cdot l = \frac{2\pi}{\lambda_0} \cdot \frac{\lambda_0}{4} = \frac{\pi}{2}$$

$$\theta \stackrel{not}{=} \beta \cdot l \quad t \stackrel{not}{=} \tan(\beta \cdot l)$$

$$Z_1^2 = Z_0 \cdot Z_L$$

# Frequency response

- matching quality = power reflection coefficient

$$\begin{aligned} |\Gamma| &= \frac{|Z_L - Z_0|}{[(Z_L + Z_0)^2 + 4t^2 Z_0 Z_L]^{1/2}} \\ &= \frac{1}{\{(Z_L + Z_0)^2/(Z_L - Z_0)^2 + [4t^2 Z_0 Z_L/(Z_L - Z_0)^2]\}^{1/2}} \\ &= \frac{1}{\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2] + [4Z_0 Z_L t^2/(Z_L - Z_0)^2]\}^{1/2}} \\ &= \frac{1}{\{1 + [4Z_0 Z_L/(Z_L - Z_0)^2] \sec^2 \theta\}^{1/2}}, \end{aligned}$$

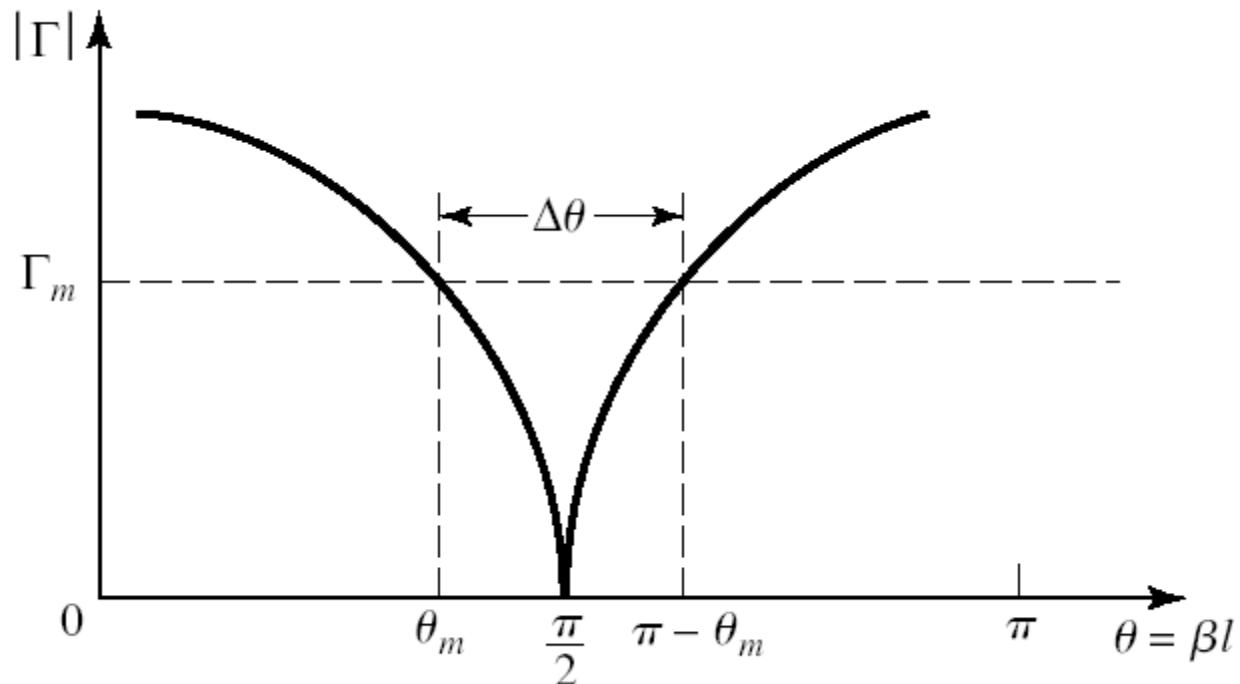
$\sec \theta = \frac{1}{\cos \theta} \rightarrow$   
 $\sec^2 \theta = 1 + \tan^2 \theta = 1 + t^2$

# Frequency response

- we assume that the operating frequency is near the design frequency (narrow bandwidth)

$$f \approx f_0 \quad l \approx \frac{\lambda_0}{4} \quad \theta \approx \frac{\pi}{2} \quad \sec^2 \theta = 1 + \tan^2 \theta \gg 1$$

$$|\Gamma| \approx \frac{|Z_L - Z_0|}{2 \cdot \sqrt{Z_0 \cdot Z_L}} \cdot |\cos \theta|$$



# Frequency response

- we set a maximum value  $\Gamma_m$  for an acceptable reflection coefficient magnitude then the bandwidth of the matching transformer,  $\theta_m$

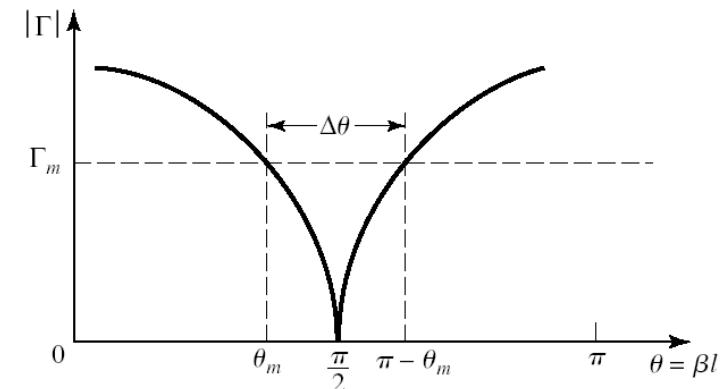
$$\frac{1}{\Gamma_m^2} = 1 + \left( \frac{2\sqrt{Z_0 Z_L}}{Z_L - Z_0} \sec \theta_m \right)^2,$$

$$\cos \theta_m = \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|}.$$

- for TEM lines

$$\theta = \beta \cdot l = \beta \cdot \frac{\lambda_0}{4} = \frac{2\pi \cdot f}{v_f} \cdot \frac{1}{4} \cdot \frac{v_f}{f_0} = \frac{\pi \cdot f}{2f_0} \quad f_m = \frac{2 \cdot \theta_m \cdot f_0}{\pi}$$

$$\frac{\Delta f}{f_0} = \frac{2 \cdot (f_0 - f_m)}{f_0} = 2 - \frac{4 \cdot \theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1} \left[ \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \cdot \frac{2\sqrt{Z_0 \cdot Z_L}}{|Z_L - Z_0|} \right]$$



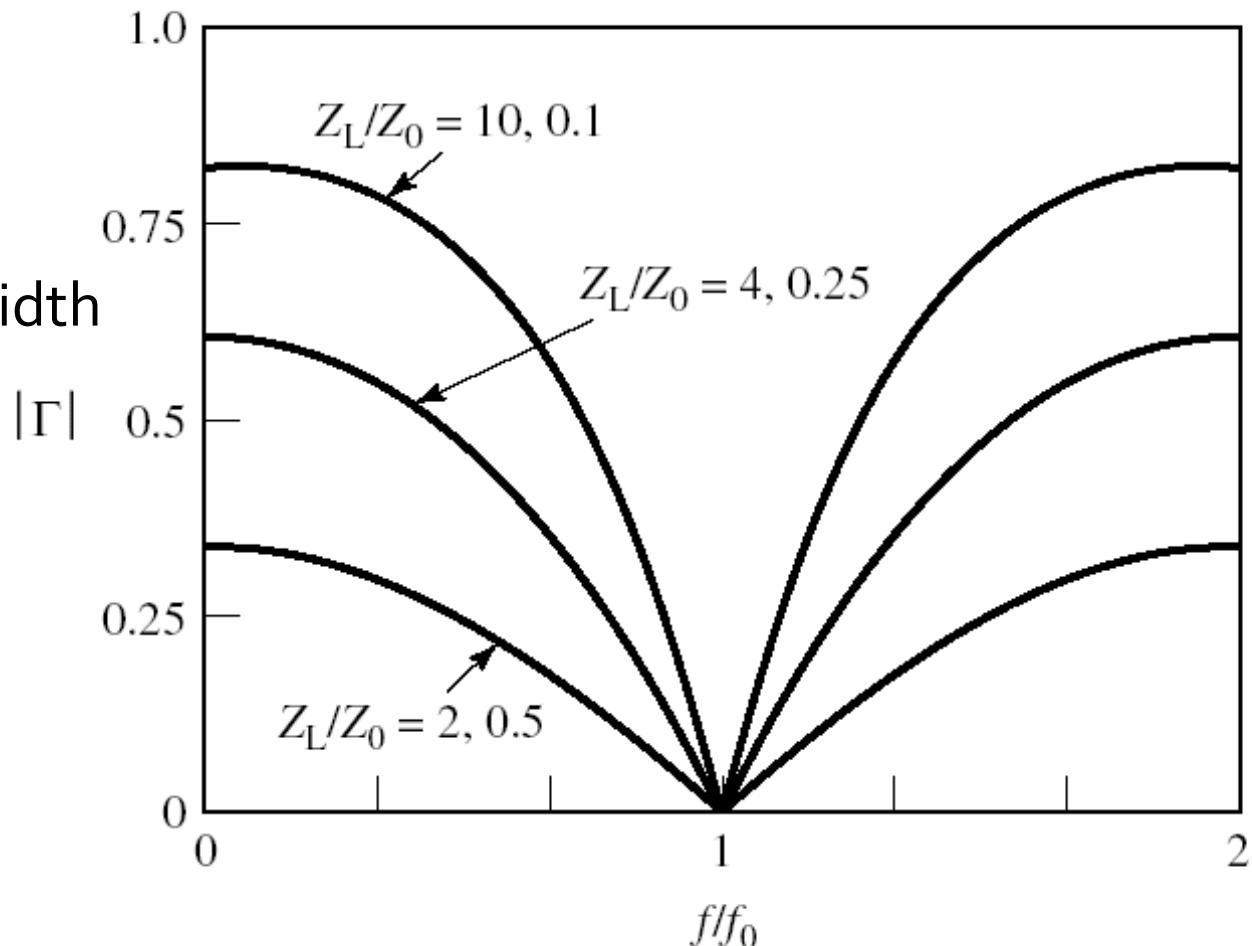
# Frequency response

- When non-TEM lines (such as waveguides) are used, the propagation constant is no longer a linear function of frequency, and the wave impedance will be frequency dependent, but in practice the bandwidth of the transformer is often small enough that these complications do not substantially affect the result
- We ignored also the effect of reactances associated with discontinuities when there is a step change in the dimensions of a transmission line ( $Z_o \rightarrow Z_1$ ). This can often be compensated by making a small adjustment in the length of the matching section

# Frequency response

- Bandwidth depends on the initial mismatch

increased bandwidth  
for smaller load  
mismatches



# Exemple

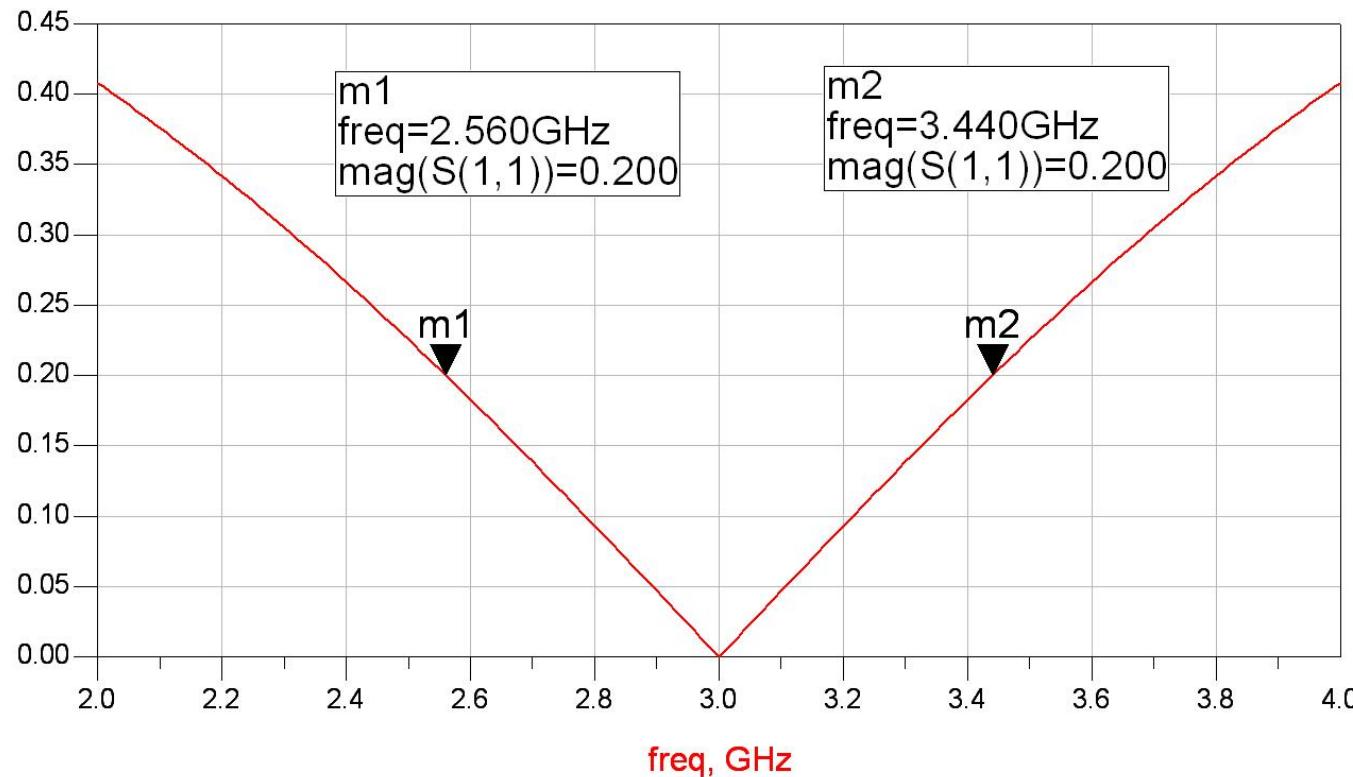
- A quarter-wave matching transformer to match a  $10\Omega$  load to a  $50\Omega$  transmission line at  $f_o=3\text{GHz}$ 
  - Determine the percent bandwidth for  $\text{SWR} < 1.5$

$$Z_1 = \sqrt{Z_0 Z_L} = \sqrt{(50)(10)} = 22.36 \Omega, \quad \Gamma_m = \frac{\text{SWR} - 1}{\text{SWR} + 1} = \frac{1.5 - 1}{1.5 + 1} = 0.2.$$

$$\begin{aligned}\frac{\Delta f}{f_0} &= 2 - \frac{4}{\pi} \cos^{-1} \left[ \frac{\Gamma_m}{\sqrt{1 - \Gamma_m^2}} \frac{2\sqrt{Z_0 Z_L}}{|Z_L - Z_0|} \right] \\ &= 2 - \frac{4}{\pi} \cos^{-1} \left[ \frac{0.2}{\sqrt{1 - (0.2)^2}} \frac{2\sqrt{(50)(10)}}{|10 - 50|} \right] \\ &= 0.29, \text{ or } 29\%.\end{aligned}$$

# Simulation

## ADS Simulation

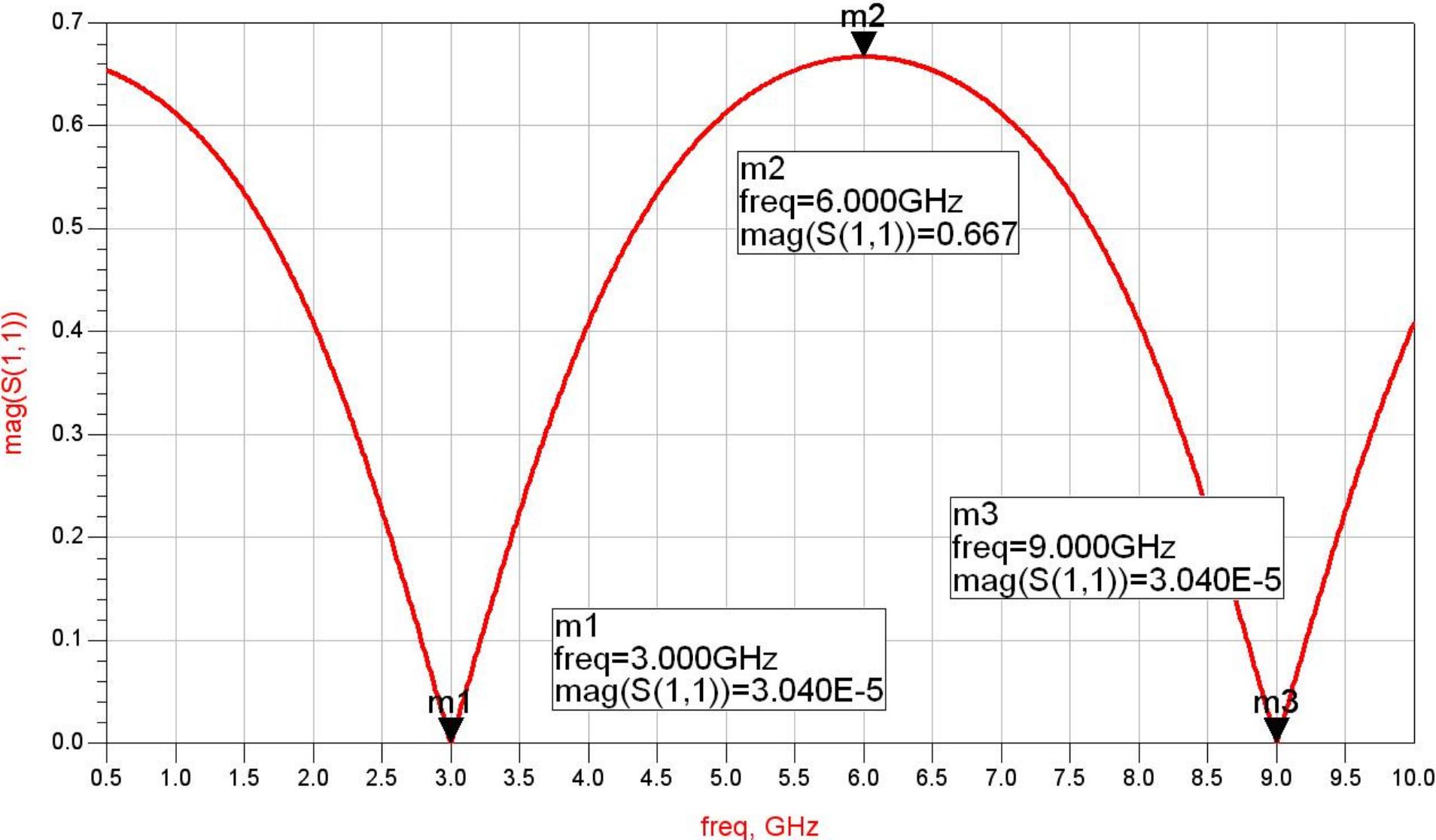


$$\Delta f = 0.88 \text{ GHz}$$

$$|\Gamma(3 \text{ GHz})| = 3 \cdot 10^{-5}$$

$$\frac{\Delta f}{f_0} = \frac{0.88}{3} = 0.2933$$

# Full bandwidth simulation



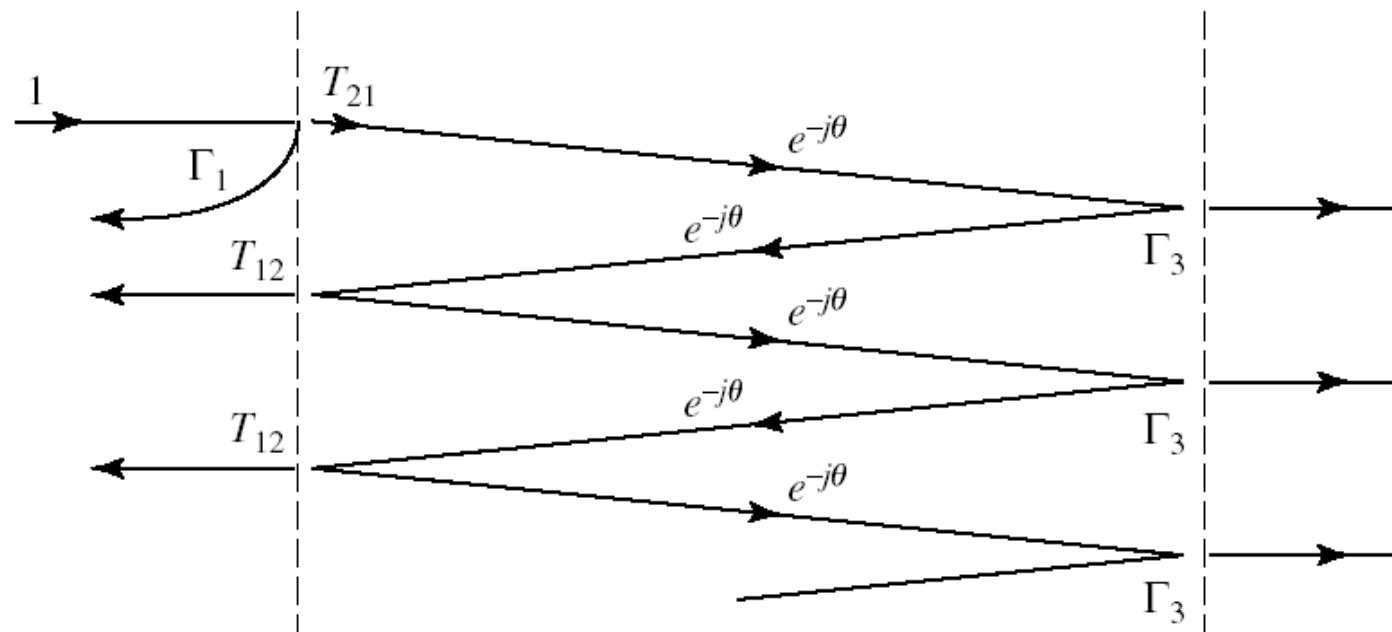
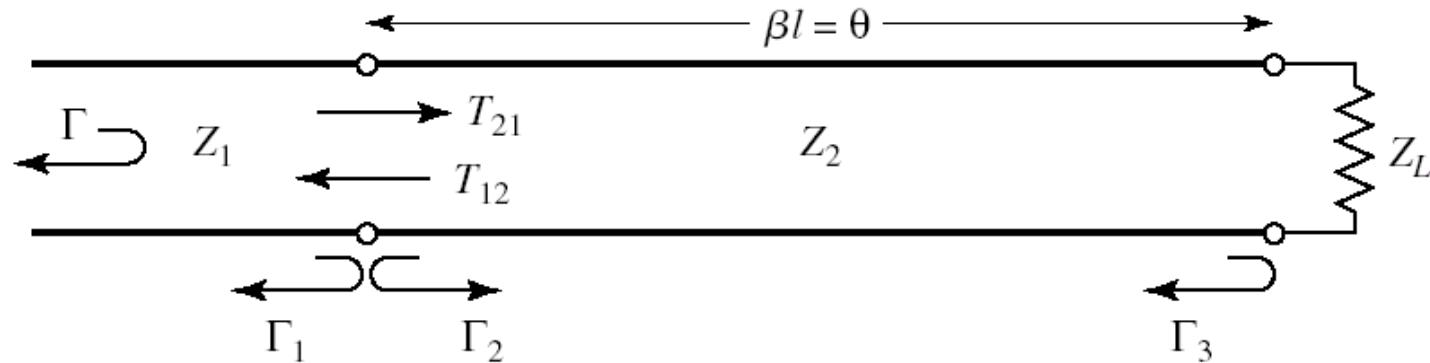
Impedance Matching with Impedance Transformers (Lab 1)

# **Impedance Matching**

# Multisection Impedance Transformer

- The quarter-wave transformer can match any real load to any feed line impedance
- If a greater bandwidth for the match is required we must use multiple sections of transmission lines transformers:
  - binomial
  - Chebyshev

# The theory of small reflections



# The theory of small reflections

$$\Gamma_1 = \frac{Z_2 - Z_1}{Z_2 + Z_1}$$

$$\Gamma_2 = -\Gamma_1$$

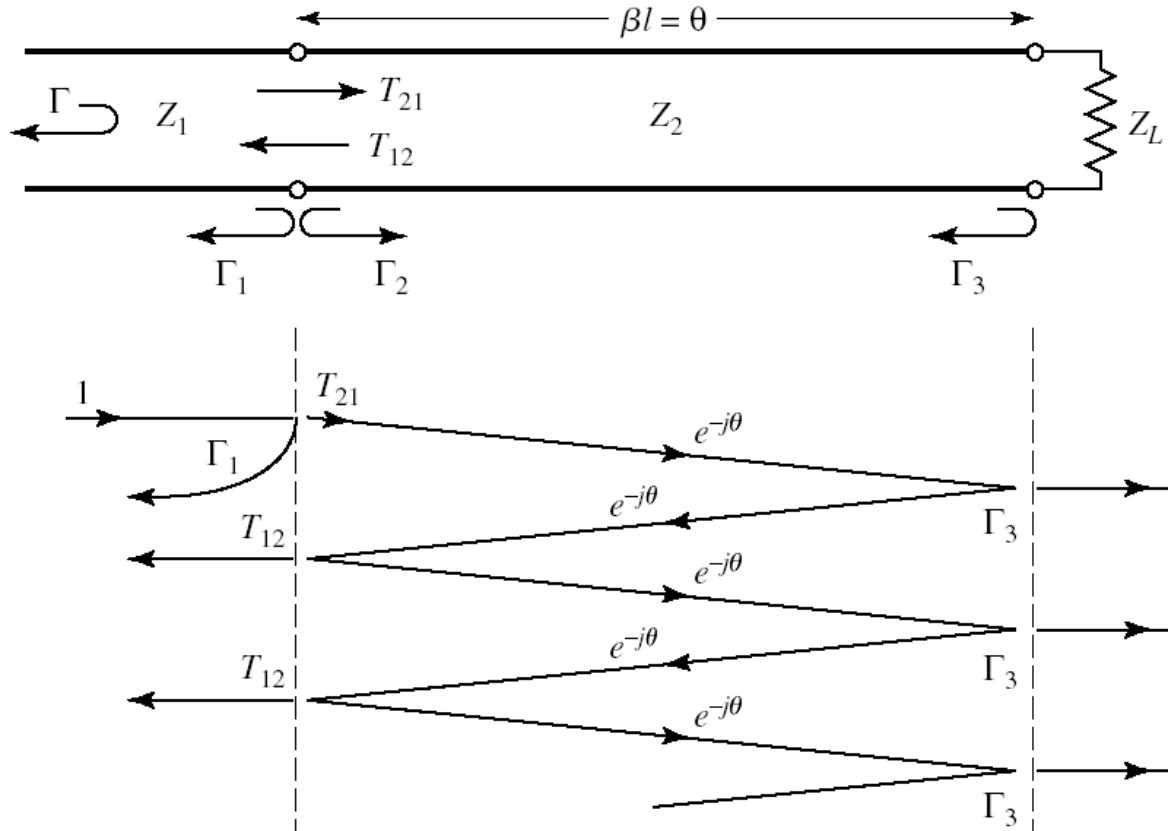
$$\Gamma_3 = \frac{Z_L - Z_2}{Z_L + Z_2}$$

$$T_{21} = 1 + \Gamma_1 = \frac{2 \cdot Z_2}{Z_1 + Z_2}$$

$$T_{12} = 1 + \Gamma_2 = \frac{2 \cdot Z_1}{Z_1 + Z_2}$$

$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^2 \cdot \Gamma_2 \cdot e^{-4j\theta} + T_{12} \cdot T_{21} \cdot \Gamma_3^3 \cdot \Gamma_2^2 \cdot e^{-6j\theta} + \dots$$

$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$



# The theory of small reflections

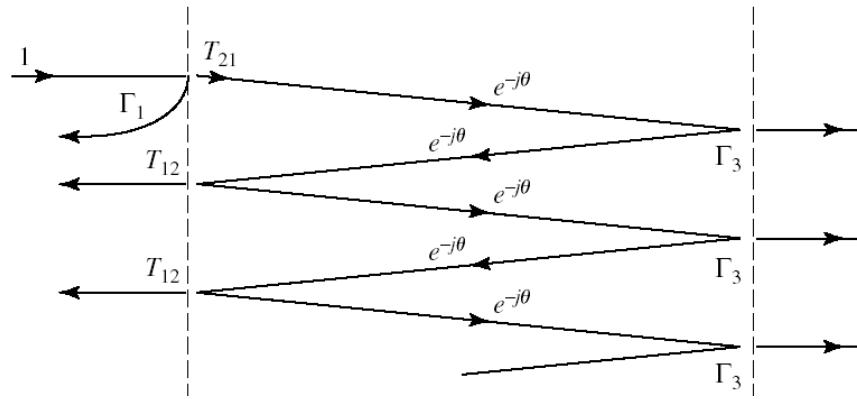
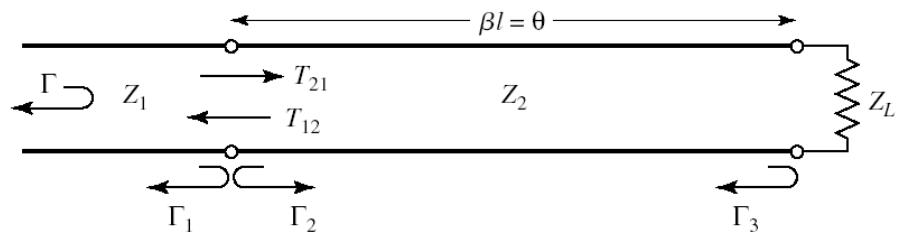
$$\Gamma = \Gamma_1 + T_{12} \cdot T_{21} \cdot \Gamma_3 \cdot e^{-2j\theta} \sum_{n=0}^{\infty} \Gamma_3^n \cdot \Gamma_2^n \cdot e^{-2jn\theta}$$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad |x| < 1$$

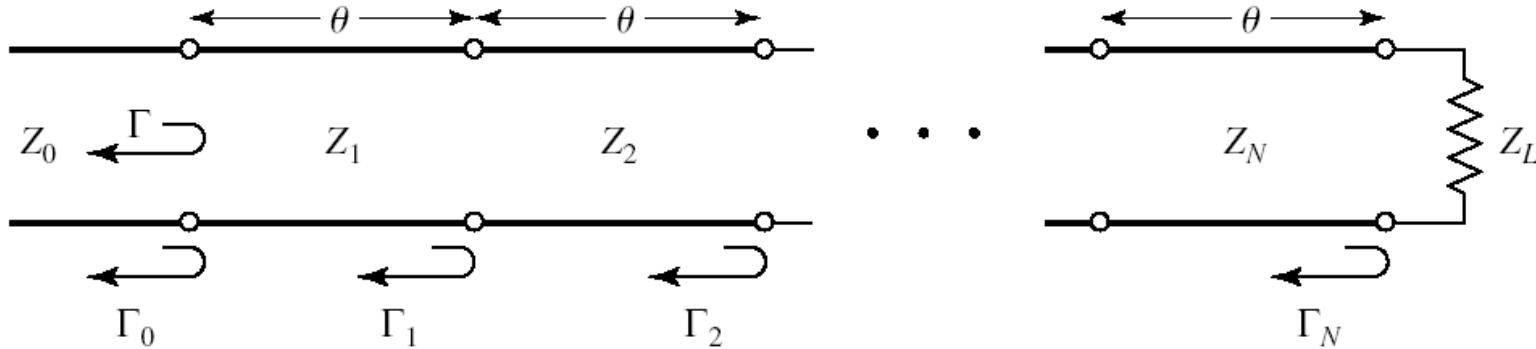
$$\Gamma = \frac{\Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}}{1 + \Gamma_1 \cdot \Gamma_3 \cdot e^{-2j\theta}}$$

- If the discontinuities between the impedances  $Z_1 \div Z_2$  and  $Z_2 \div Z_L$  are small we can approximate

$$\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta}$$



# Multisection transformers



- We also assume that all impedances **increase or decrease monotonically** across the transformer
- This implies that all reflection coefficients will be real and of the same sign
- Previously, 1 section  $\Gamma \cong \Gamma_1 + \Gamma_3 \cdot e^{-2j\theta} \Rightarrow \Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$

$$\Gamma_1 = \frac{Z_1 - Z_0}{Z_1 + Z_0}$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n}$$

$$n = \overline{1, N-1}$$

$$\Gamma_N = \frac{Z_L - Z_N}{Z_L + Z_N}$$

# Multisection transformers

- assume that the transformer can be made **symmetrical**

$$\Gamma_0 = \Gamma_N, \Gamma_1 = \Gamma_{N-1}, \Gamma_2 = \Gamma_{N-2} \dots$$

- Note that this does **not** imply that the impedances are symmetrical

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \dots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma(\theta) = e^{-jN\theta} \cdot [\Gamma_0 \cdot (e^{jN\theta} + e^{-jN\theta}) + \Gamma_1 \cdot (e^{j(N-2)\theta} + e^{-j(N-2)\theta}) + \Gamma_2 \cdot (e^{j(N-4)\theta} + e^{-j(N-4)\theta}) + \dots]$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots + \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

last item:       $\cdots \frac{1}{2} \cdot \Gamma_{N/2}$      $N$  even                         $\cdots \Gamma_{(N-1)/2} \cdot \cos \theta$      $N$  odd

# Multisection transformers

- Input reflection coefficient

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \cdots + a_N \cdot x^N$$

- we can choose the coefficients so we obtain a desired behavior (of the polynomial)

# Binomial multisection transformer

- The response is as flat as possible near the design frequency, also known as **maximally flat**
- For N sections the first N-1 derivatives of the  $|\Gamma(\theta)|$  functions are annulled

$$f(x) = A \cdot (1+x)^N$$

$$\Gamma(\theta) = A \cdot (1 + e^{-2j\theta})^N$$

$$|\Gamma(\theta)| = |A| \cdot |e^{-j\theta}|^N \cdot |e^{j\theta} + e^{-j\theta}|^N = 2^N \cdot |A| \cdot |\cos\theta|^N$$

$$l = \frac{\lambda}{4} \Rightarrow \theta = \beta \cdot l = \frac{\pi}{2} \quad \left| \Gamma\left(\frac{\pi}{2}\right) \right| = 0; \quad \frac{d^n}{d\theta^n} |\Gamma(\theta)|_{\theta=\frac{\pi}{2}} = 0 \quad n = \overline{1, N-1}$$

# Binomial multisection transformer

- $A, \theta \rightarrow 0$ ,  $0$  length sections, the sections disappear

$$\Gamma(0) = 2^N \cdot A = \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

- Binomial expansion

$$f(x) = (1+x)^N = C_N^0 + C_N^1 \cdot x + \cdots + C_N^n \cdot x^n + \cdots + C_N^N \cdot x^N$$

$$C_N^n = \frac{N!}{(N-n)!n!}$$

- Reflection coefficient:

$$\Gamma(\theta) = A \cdot (1 + e^{-2j\theta})^N \quad \Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$\Gamma_n = A \cdot C_N^n$$

# Binomial multisection transformer

## ■ Manual design procedure

$$A = 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0}$$

$$\Gamma_n = A \cdot C_N^n$$

$$\Gamma_n = \frac{Z_{n+1} - Z_n}{Z_{n+1} + Z_n} \cong \frac{1}{2} \ln \frac{Z_{n+1}}{Z_n}$$

$$\ln x \cong 2 \cdot \frac{x-1}{x+1} \quad x \cong 1$$

$$\ln \frac{Z_{n+1}}{Z_n} \cong 2 \cdot \Gamma_n = 2 \cdot A \cdot C_N^n = 2 \cdot 2^{-N} \cdot \frac{Z_L - Z_0}{Z_L + Z_0} \cong 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

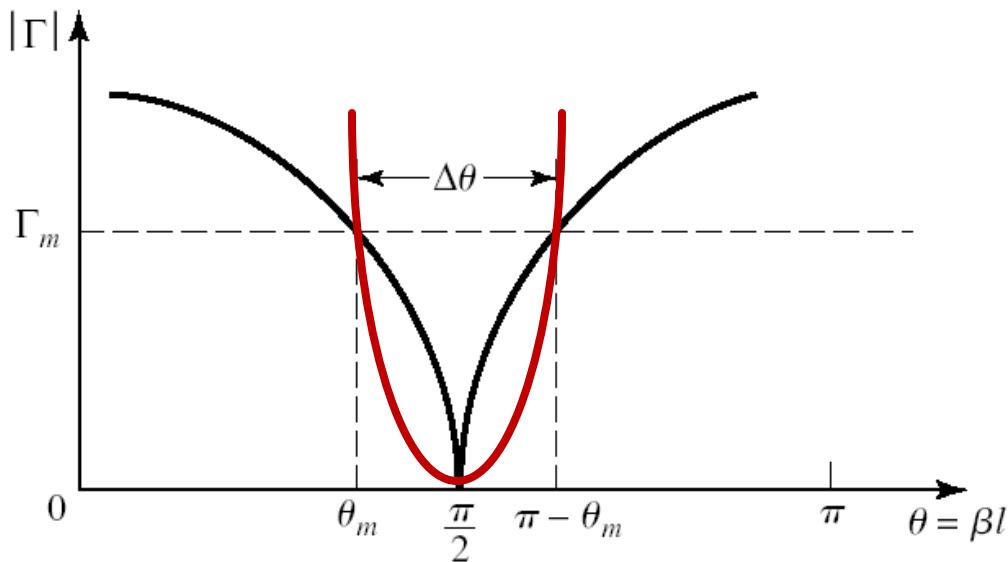
$$\ln Z_{n+1} \cong \ln Z_n + 2^{-N} \cdot C_N^n \cdot \ln \frac{Z_L}{Z_0}$$

# Binomial multisection transformer

- Bandwidth,  $\Gamma_m$  maximum acceptable value

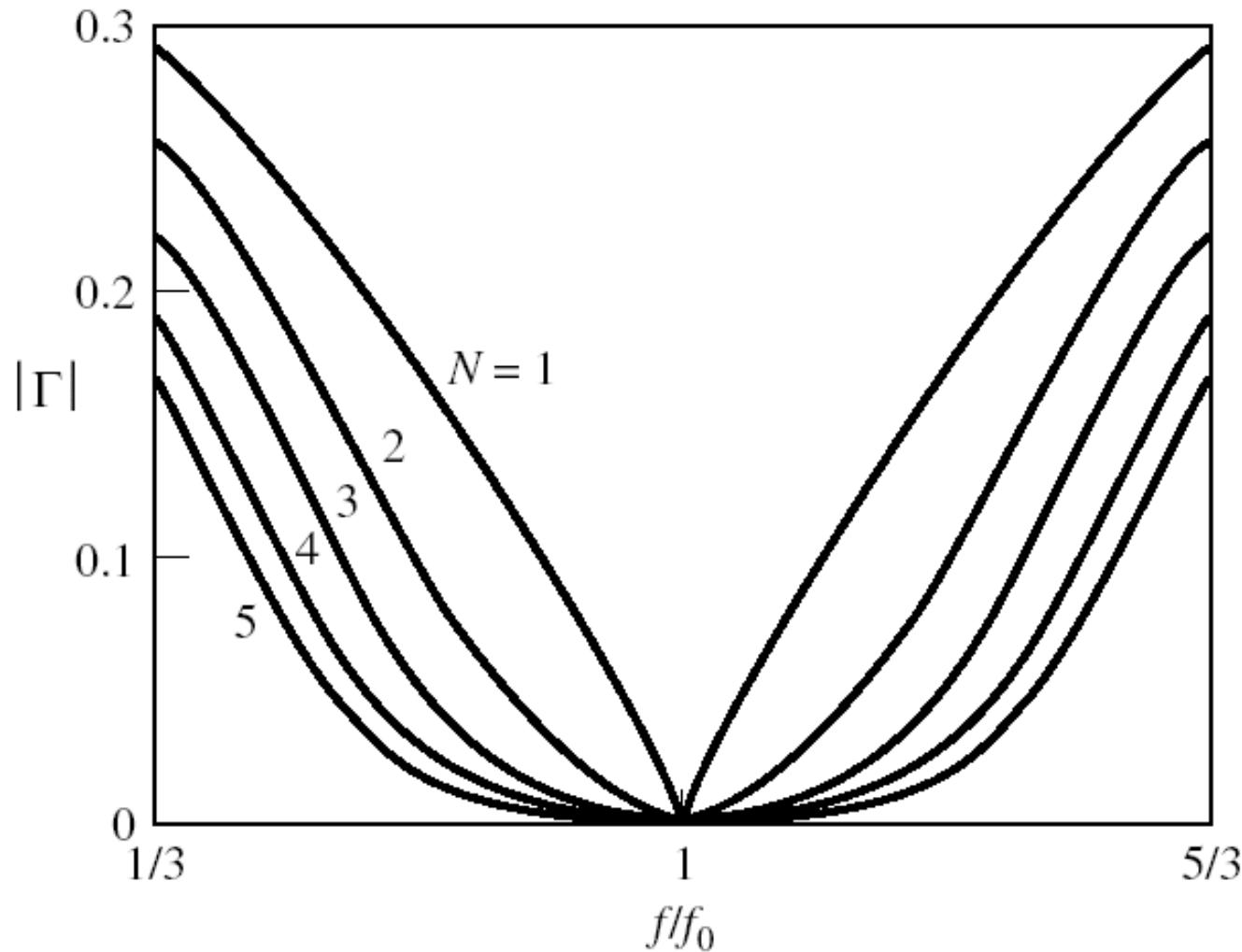
$$\Gamma_m = |\Gamma(\theta_m)| = 2^N \cdot |A| \cdot |\cos \theta_m|^N$$

$$\theta_m = \cos^{-1} \left[ \frac{1}{2} \left( \frac{\Gamma_m}{|A|} \right)^{\frac{1}{N}} \right]$$



$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4}{\pi} \cdot \cos^{-1} \left[ \frac{1}{2} \left( \frac{\Gamma_m}{|A|} \right)^{\frac{1}{N}} \right]$$

# Bandwidth



# Binomial multisection transformer

## Exact results

$Z_L/Z_0$	$N = 2$		$N = 3$			$N = 4$					
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$		
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
1.5	1.1067	1.3554	1.0520	1.2247	1.4259	1.0257	1.1351	1.3215	1.4624		
2.0	1.1892	1.6818	1.0907	1.4142	1.8337	1.0444	1.2421	1.6102	1.9150		
3.0	1.3161	2.2795	1.1479	1.7321	2.6135	1.0718	1.4105	2.1269	2.7990		
4.0	1.4142	2.8285	1.1907	2.0000	3.3594	1.0919	1.5442	2.5903	3.6633		
6.0	1.5651	3.8336	1.2544	2.4495	4.7832	1.1215	1.7553	3.4182	5.3500		
8.0	1.6818	4.7568	1.3022	2.8284	6.1434	1.1436	1.9232	4.1597	6.9955		
10.0	1.7783	5.6233	1.3409	3.1623	7.4577	1.1613	2.0651	4.8424	8.6110		
$Z_L/Z_0$	$N = 5$					$N = 6$					
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$	$Z_5/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$	$Z_5/Z_0$	$Z_6/Z_0$
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.0128	1.0790	1.2247	1.3902	1.4810	1.0064	1.0454	1.1496	1.3048	1.4349	1.4905
2.0	1.0220	1.1391	1.4142	1.7558	1.9569	1.0110	1.0790	1.2693	1.5757	1.8536	1.9782
3.0	1.0354	1.2300	1.7321	2.4390	2.8974	1.0176	1.1288	1.4599	2.0549	2.6577	2.9481
4.0	1.0452	1.2995	2.0000	3.0781	3.8270	1.0225	1.1661	1.6129	2.4800	3.4302	3.9120
6.0	1.0596	1.4055	2.4495	4.2689	5.6625	1.0296	1.2219	1.8573	3.2305	4.9104	5.8275
8.0	1.0703	1.4870	2.8284	5.3800	7.4745	1.0349	1.2640	2.0539	3.8950	6.3291	7.7302
10.0	1.0789	1.5541	3.1623	6.4346	9.2687	1.0392	1.2982	2.2215	4.5015	7.7030	9.6228

# Exemple

- Design a three-section binomial transformer to match a  $30\Omega$  load to a  $100 \Omega$  line at  $f_o=3\text{GHz}$ ,  $\Gamma_m=0.1$ 
  - $N = 3$

$$Z_L = 30\Omega \quad Z_0 = 100 \Omega$$

$$A = 2^{-N} \frac{Z_L - Z_0}{Z_L + Z_0} \approx \frac{1}{2^{N+1}} \ln \frac{Z_L}{Z_0} = -0.07525$$

$$C_3^0 = \frac{3!}{3! \cdot 0!} = 1 \quad C_3^1 = \frac{3!}{2! \cdot 1!} = 3 \quad C_3^2 = \frac{3!}{1! \cdot 2!} = 3$$

# Exemple

$$n = 0$$

$$\ln Z_1 = \ln Z_0 + 2^{-N} C_3^0 \ln \frac{Z_L}{Z_0} = \ln 100 + 2^{-3} \cdot 1 \cdot \ln \frac{30}{100} = 4.455$$

$$Z_1 = 86.03 \Omega$$

$$n = 1$$

$$\ln Z_2 = \ln Z_1 + 2^{-N} C_3^1 \ln \frac{Z_L}{Z_0} = \ln 86.03 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 4.003$$

$$Z_2 = 54.77 \Omega$$

$$n = 2$$

$$\ln Z_3 = \ln Z_2 + 2^{-N} C_3^2 \ln \frac{Z_L}{Z_0} = \ln 54.77 + 2^{-3} \cdot 3 \cdot \ln \frac{30}{100} = 3.552$$

$$Z_3 = 34.87 \Omega$$

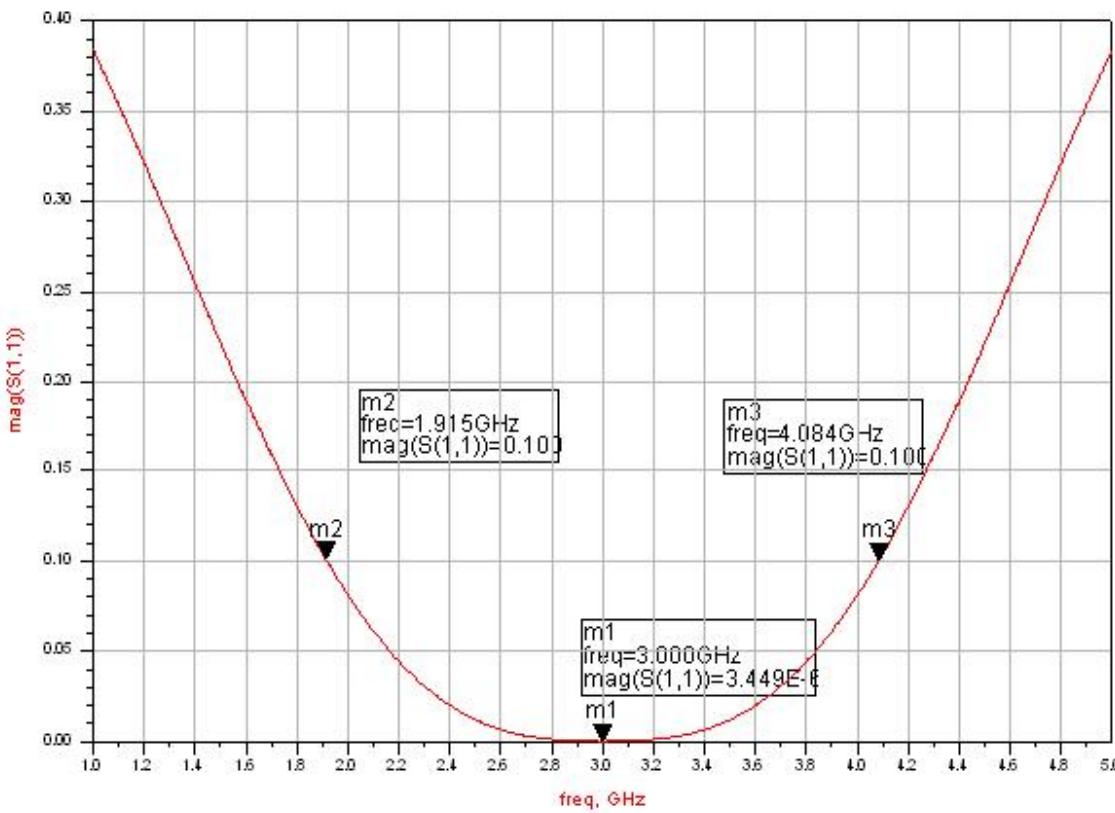
# Exemple

$$\frac{\Delta f}{f_0} = 2 - \frac{4}{\pi} \arccos \left[ \frac{1}{2} \left( \frac{\Gamma_m}{|A|} \right)^{1/N} \right] = 2 - \frac{4}{\pi} \arccos \left[ \frac{1}{2} \left( \frac{0.1}{0.07525} \right)^{1/3} \right] = 0.74$$

$$\Delta f = 2.22 \text{GHz}$$

# Simulation

## ■ Similarly Lab. 1



$$\Delta f = 2.169 \text{ GHz}$$

$$|\Gamma(3 \text{ GHz})| = 3.5 \cdot 10^{-6}$$

# Chebyshev multisection transformer

- The response of this multisection impedance transformer is equal-ripple in passband
- optimizes (increases) bandwidth at the expense of passband ripple
- We match the  $\Gamma(\theta)$  function with an desired Chebyshev polynomial

# Chebyshev polynomials

$$T_1(x) = x$$

$$T_2(x) = 2x^2 - 1$$

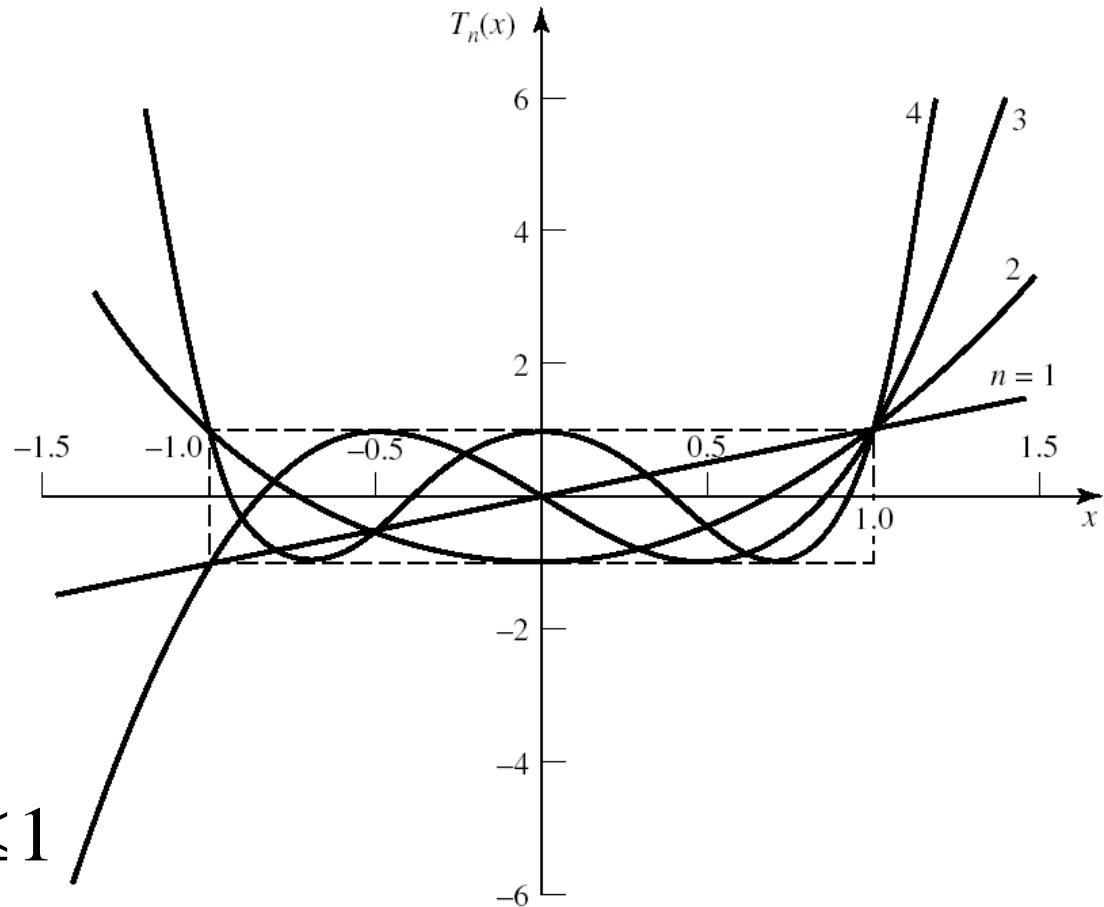
$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

■ equal-ripple

$$-1 \leq x \leq 1 \Rightarrow |T_n(x)| \leq 1$$

$$T_n(x) = 2xT_{n-1}(x) - T_{n-2}(x)$$



# Chebyshev polynomials

$$T_1(x) = x$$

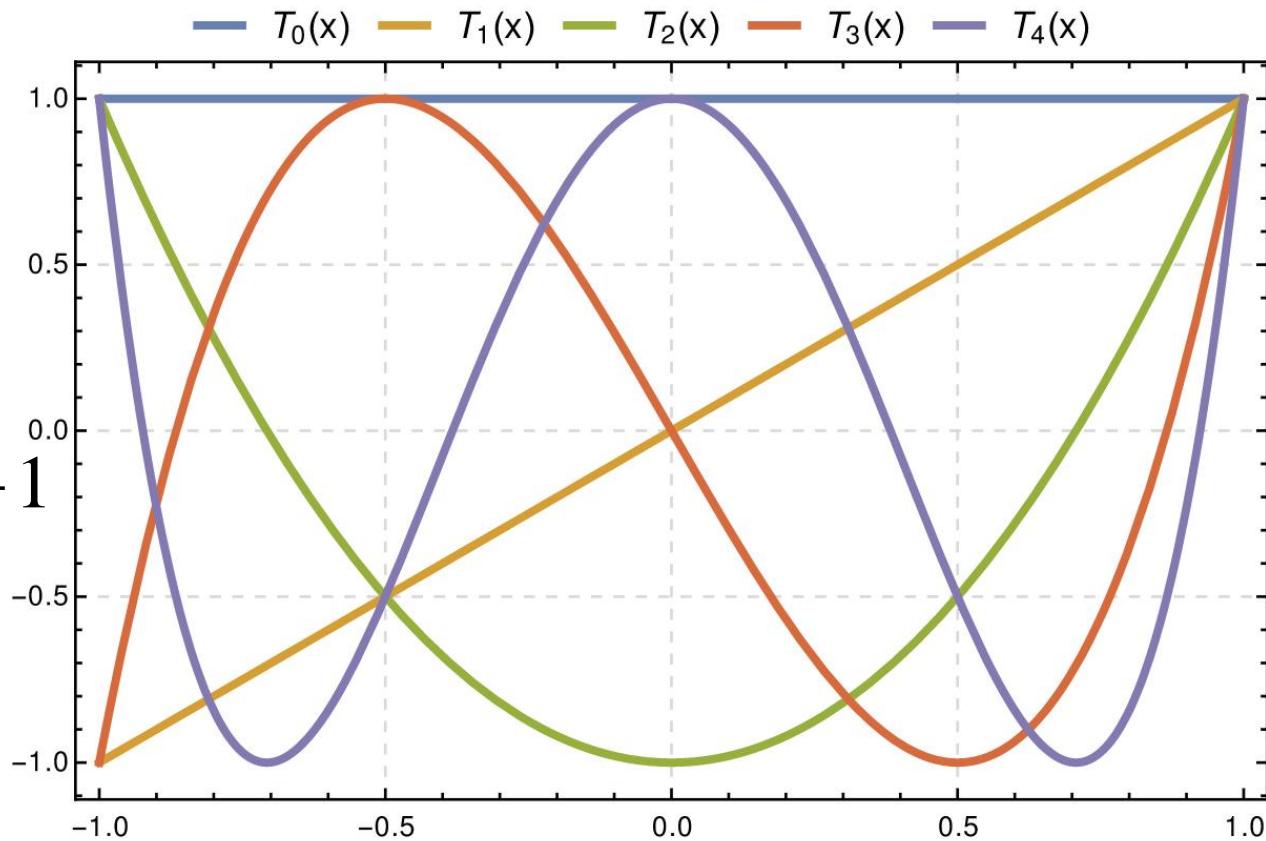
$$T_2(x) = 2x^2 - 1$$

$$T_3(x) = 4x^3 - 3x$$

$$T_4(x) = 8x^4 - 8x^2 + 1$$

■ equal-ripple

$$-1 \leq x \leq 1 \Rightarrow |T_n(x)| \leq 1$$



# Chebyshev polynomials

$$\Gamma(\theta) = \Gamma_0 + \Gamma_1 \cdot e^{-2j\theta} + \Gamma_2 \cdot e^{-4j\theta} + \cdots + \Gamma_N \cdot e^{-2jN\theta}$$

$$e^{-2j\theta} \equiv x$$

$$f(x) = a_0 + a_1 \cdot x + a_2 \cdot x^2 + \cdots + a_N \cdot x^N$$

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \cdots + \Gamma_n \cdot \cos(N-2n)\theta + \cdots]$$

last item:  $\cdots \frac{1}{2} \cdot \Gamma_{N/2} \quad N \text{ even}$

$$\cdots \Gamma_{(N-1)/2} \cdot \cos \theta \quad N \text{ odd}$$

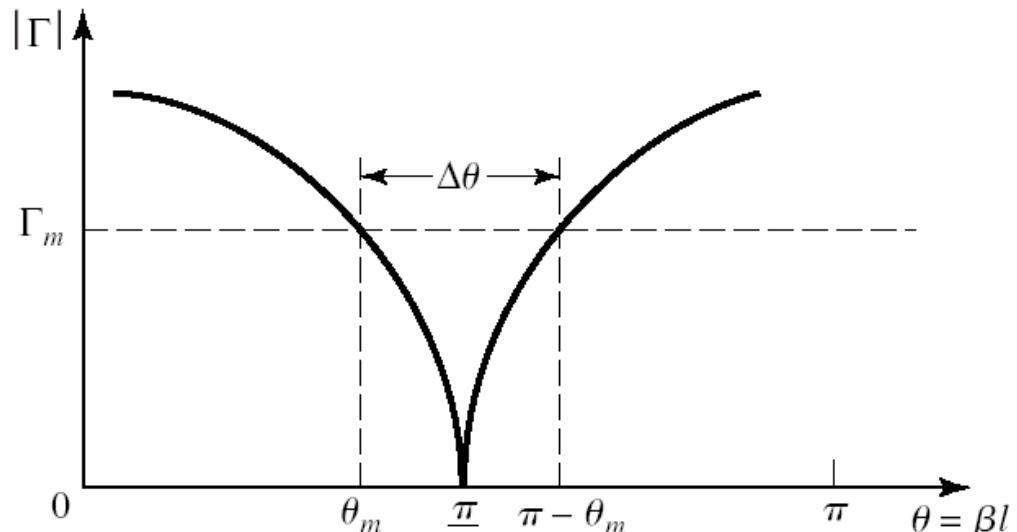
$$x = \cos \theta \quad |x| < 1$$

- We can show that:  $T_n(\cos \theta) = \cos(n\theta)$

$$T_n(x) = \cos(n \arccos(x)) \quad |x| < 1 \quad T_n(x) = \cosh(n \cosh^{-1}(x)) \quad |x| > 1$$

# Chebyshev multisection transformer

- variable change  
so we map:
  - bandwidth  $\rightarrow [-1, 1]$



$$\theta = \theta_m \rightarrow x = 1$$

$$\theta = \pi - \theta_m \rightarrow x = -1$$

$$x \equiv \frac{\cos \theta}{\cos \theta_m}$$

$$\sec \theta = \frac{1}{\cos \theta}$$

$$x = \sec \theta_m \cos \theta$$

# Chebyshev multisection transformer

- We search the coefficients to obtain a Chebyshev polynomial

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots + \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

ultimul termen:  
 $\dots \frac{1}{2} \cdot \Gamma_{N/2}$      $n$  par

$\dots \Gamma_{(N-1)/2} \cdot \cos \theta$      $n$  impar


$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N(\sec \theta_m \cos \theta)$$

# Chebyshev polynomials

$$\cancel{(\cos\theta)^k} \Leftrightarrow \cos k\theta$$

$$T_1(x) = x$$

$$T_1(\sec\theta_m \cos\theta) = \sec\theta_m \cos\theta$$

$$T_2(x) = 2x^2 - 1$$

$$T_2(\sec\theta_m \cos\theta) = 2\sec^2\theta_m \cos^2\theta - 1$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1$$

$$T_2(\sec\theta_m \cos\theta) = \sec^2\theta_m (1 + \cos 2\theta) - 1$$

$$T_3(x) = 4x^3 - 3x \quad T_3(\sec\theta_m \cos\theta) = 4\sec^3\theta_m \cos^3\theta - 3\sec\theta_m \cos\theta$$

$$\cos 3\theta = \cos(2\theta + \theta) = \cos 2\theta \cos\theta - \sin 2\theta \sin\theta$$

$$\cos 3\theta = (2\cos^2\theta - 1)\cos\theta - 2(1 - \cos^2\theta)\cos\theta$$

$$\cos 3\theta = 4\cos^3\theta - 3\cos\theta$$

$$T_3(\sec\theta_m \cos\theta) = \sec^3\theta_m (\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$$

# Chebyshev multisection transformer

$$T_1(\sec\theta_m \cos\theta) = \sec\theta_m \cos\theta$$

$$T_2(\sec\theta_m \cos\theta) = \sec^2 \theta_m (1 + \cos 2\theta) - 1$$

$$T_3(\sec\theta_m \cos\theta) = \sec^3 \theta_m (\cos 3\theta + 3\cos\theta) - 3\sec\theta_m \cos\theta$$

$$T_4(\sec\theta_m \cos\theta) = \sec^4 \theta_m (\cos 4\theta + 4\cos 2\theta + 3) - 4\sec^2 \theta_m (\cos 2\theta + 1) + 1$$

- We search coefficients of  $\Gamma(\theta)$  function to obtain a Chebyshev polynomial

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots + \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

$$\Gamma(\theta) = A \cdot e^{-jN\theta} \cdot T_N(\sec\theta_m \cos\theta)$$

last item:  $\cdots \frac{1}{2} \cdot \Gamma_{N/2}$      $N$  even

$$\cdots \Gamma_{(N-1)/2} \cdot \cos\theta \quad N \text{ odd}$$

# Chebyshev multisection transformer

- $A, \theta \rightarrow 0$ , 0 length sections, the sections disappear

$$\Gamma(0) = \frac{Z_L - Z_0}{Z_L + Z_0} = A \cdot T_N(\sec \theta_m) \quad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec \theta_m)} \quad \boxed{\Gamma_m = |A|}$$

$$T_N(\sec \theta_m) = \frac{1}{\Gamma_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \approx \frac{1}{2\Gamma_m} \left| \ln \frac{Z_L}{Z_0} \right|$$
$$T_n(x) = \cosh(n \cosh^{-1}(x))$$

$$\sec \theta_m = \cosh \left[ \frac{1}{N} \cosh^{-1} \left( \frac{1}{\Gamma_m} \left| \frac{Z_L - Z_0}{Z_L + Z_0} \right| \right) \right] \approx \cosh \left[ \frac{1}{N} \cosh^{-1} \left( \left| \frac{\ln(Z_L/Z_0)}{2\Gamma_m} \right| \right) \right]$$

- we compute  $\theta_m$  for maximum acceptable value  $\Gamma_m$  (ripple) then bandwidth is:

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi}$$

# Chebyshev multisection transformer

- Design procedure, approximate solutions

$$\Gamma_m = |A|$$

$$A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec \theta_m)}$$

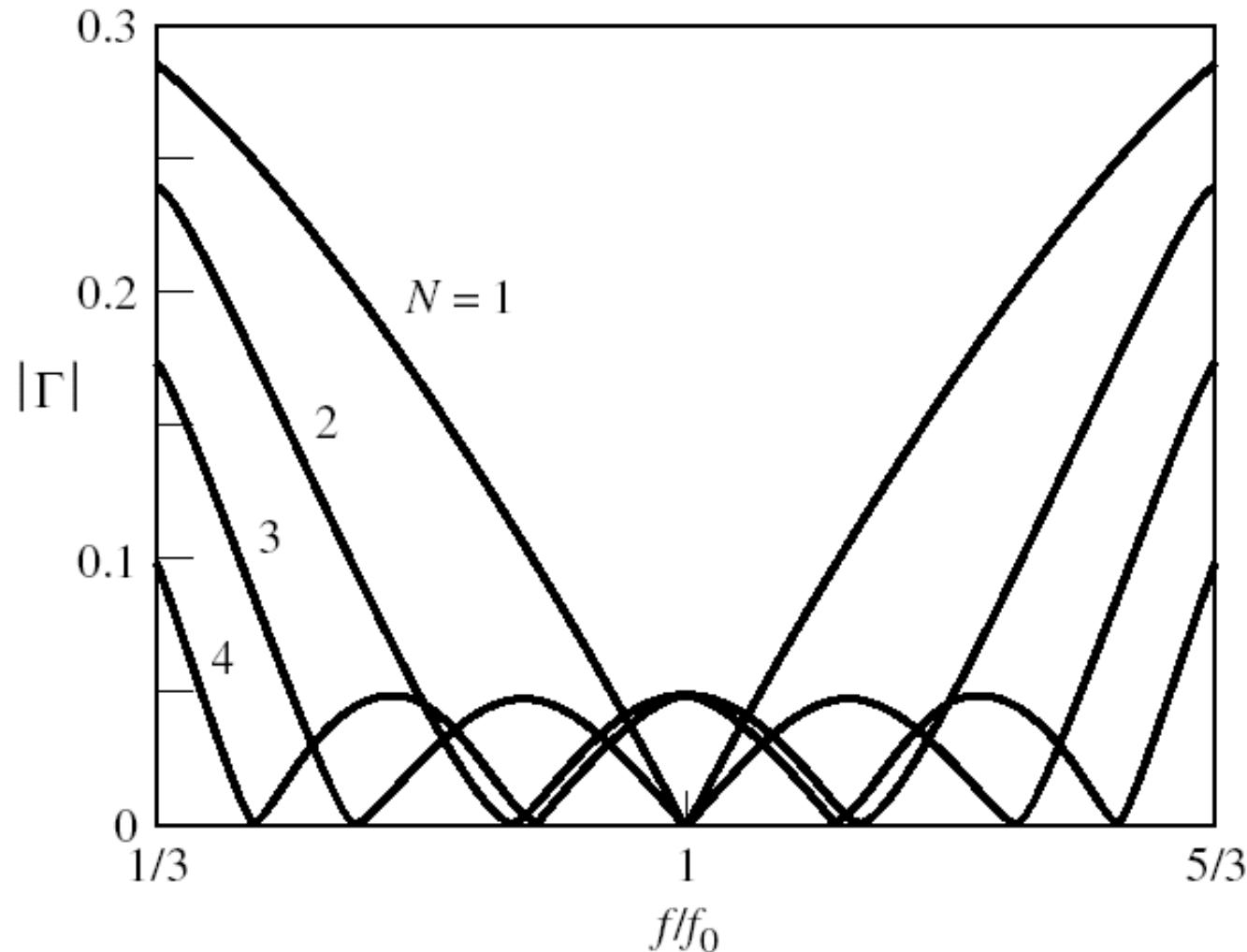
- Sign of A depends on  $Z_L <> Z_0$
- Compute  $\sec \theta_m$
- Write down the Chebyshev polynomial for the order of your choice and identify  $\cos k\theta$  coefficients

$$\Gamma(\theta) = 2e^{-jN\theta} \cdot [\Gamma_0 \cdot \cos N\theta + \Gamma_1 \cdot \cos(N-2)\theta + \dots + \Gamma_n \cdot \cos(N-2n)\theta + \dots]$$

$$\ln \frac{Z_{n+1}}{Z_n} \approx 2 \cdot \Gamma_n$$

$$\ln Z_{n+1} \approx \ln Z_n + 2 \cdot \Gamma_n$$

# Bandwidth



# Chebyshev multisection transformer

## Exact results

$Z_L/Z_0$	$N = 2$				$N = 3$					
	$\Gamma_m = 0.05$		$\Gamma_m = 0.20$		$\Gamma_m = 0.05$			$\Gamma_m = 0.20$		
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1.5	1.1347	1.3219	1.2247	1.2247	1.1029	1.2247	1.3601	1.2247	1.2247	1.2247
2.0	1.2193	1.6402	1.3161	1.5197	1.1475	1.4142	1.7429	1.2855	1.4142	1.5558
3.0	1.3494	2.2232	1.4565	2.0598	1.2171	1.7321	2.4649	1.3743	1.7321	2.1829
4.0	1.4500	2.7585	1.5651	2.5558	1.2662	2.0000	3.1591	1.4333	2.0000	2.7908
6.0	1.6047	3.7389	1.7321	3.4641	1.3383	2.4495	4.4833	1.5193	2.4495	3.9492
8.0	1.7244	4.6393	1.8612	4.2983	1.3944	2.8284	5.7372	1.5766	2.8284	5.0742
10.0	1.8233	5.4845	1.9680	5.0813	1.4385	3.1623	6.9517	1.6415	3.1623	6.0920
$N = 4$										
$Z_L/Z_0$	$\Gamma_m = 0.05$				$\Gamma_m = 0.20$					
	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$	$Z_1/Z_0$	$Z_2/Z_0$	$Z_3/Z_0$	$Z_4/Z_0$		
1.0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000		
1.5	1.0892	1.1742	1.2775	1.3772	1.2247	1.2247	1.2247	1.2247		
2.0	1.1201	1.2979	1.5409	1.7855	1.2727	1.3634	1.4669	1.5715		
3.0	1.1586	1.4876	2.0167	2.5893	1.4879	1.5819	1.8965	2.0163		
4.0	1.1906	1.6414	2.4369	3.3597	1.3692	1.7490	2.2870	2.9214		
6.0	1.2290	1.8773	3.1961	4.8820	1.4415	2.0231	2.9657	4.1623		
8.0	1.2583	2.0657	3.8728	6.3578	1.4914	2.2428	3.5670	5.3641		
10.0	1.2832	2.2268	4.4907	7.7930	1.5163	2.4210	4.1305	6.5950		

# Exemple

- Design a three-section Chebyshev transformer to match a  $30\Omega$  load to a  $100\Omega$  line at  $f_o=3\text{GHz}$ ,  $\Gamma_m=0.1$ 
  - $N = 3 \quad Z_L = 30\Omega \quad Z_0 = 100\Omega$

$$\Gamma(\theta) = 2e^{-j3\theta} [\Gamma_0 \cos 3\theta + \Gamma_1 \cos \theta] = A e^{-j3\theta} T_3(\sec \theta_m \cos \theta)$$

$$|A| = \Gamma_m = 0.1 \quad A = \frac{Z_L - Z_0}{Z_L + Z_0} \cdot \frac{1}{T_N(\sec \theta_m)} \quad Z_L < Z_0 \rightarrow A < 0 \quad A = -0.1$$

$$\sec \theta_m = \cosh \left[ \frac{1}{N} \cdot \cosh^{-1} \left( \left| \frac{\ln Z_L / Z_0}{2\Gamma_m} \right| \right) \right] = \cosh \left[ \frac{1}{3} \cdot \cosh^{-1} \left( \left| \frac{\ln(30/100)}{2 \cdot 0.1} \right| \right) \right] = 1.362$$

$$\theta_m = \arccos \left( \frac{1}{\sec \theta_m} \right) = 0.746 \text{ rad} = 42.76^\circ$$

# Exemple

$$2[\Gamma_0 \cos 3\theta + \Gamma_1 \cos \theta] = A \sec^3 \theta_m (\cos 3\theta + 3 \cos \theta) - 3A \sec \theta_m \cos \theta$$

$$\cos 3\theta$$

$$\cos \theta$$

$$2\Gamma_0 = A \sec^3 \theta_m$$

$$2\Gamma_1 = 3A(\sec^3 \theta_m - \sec \theta_m)$$

$$\Gamma_0 = -0.1263$$

$$\Gamma_1 = -0.1747$$

simetrie:  $\Gamma_3 = \Gamma_0; \quad \Gamma_2 = \Gamma_1$

# Exemple

$n = 0$

$$\ln Z_1 = \ln Z_0 + 2 \cdot \Gamma_0 = \ln 100 - 2 \cdot 0.1263 = 4.353 \quad \Gamma_0 = -0.1263$$

$$Z_1 = 77.68\Omega \quad \Gamma_1 = -0.1747$$

$n = 1$

$$\ln Z_2 = \ln Z_1 + 2 \cdot \Gamma_1 = \ln 77.68 - 2 \cdot 0.1747 = 4.003$$

$$Z_2 = 54.77\Omega$$

$n = 2$

$$\ln Z_3 = \ln Z_2 + 2 \cdot \Gamma_2 = \ln 54.77 - 2 \cdot 0.1747 = 3.654$$

$$Z_3 = 38.62\Omega$$

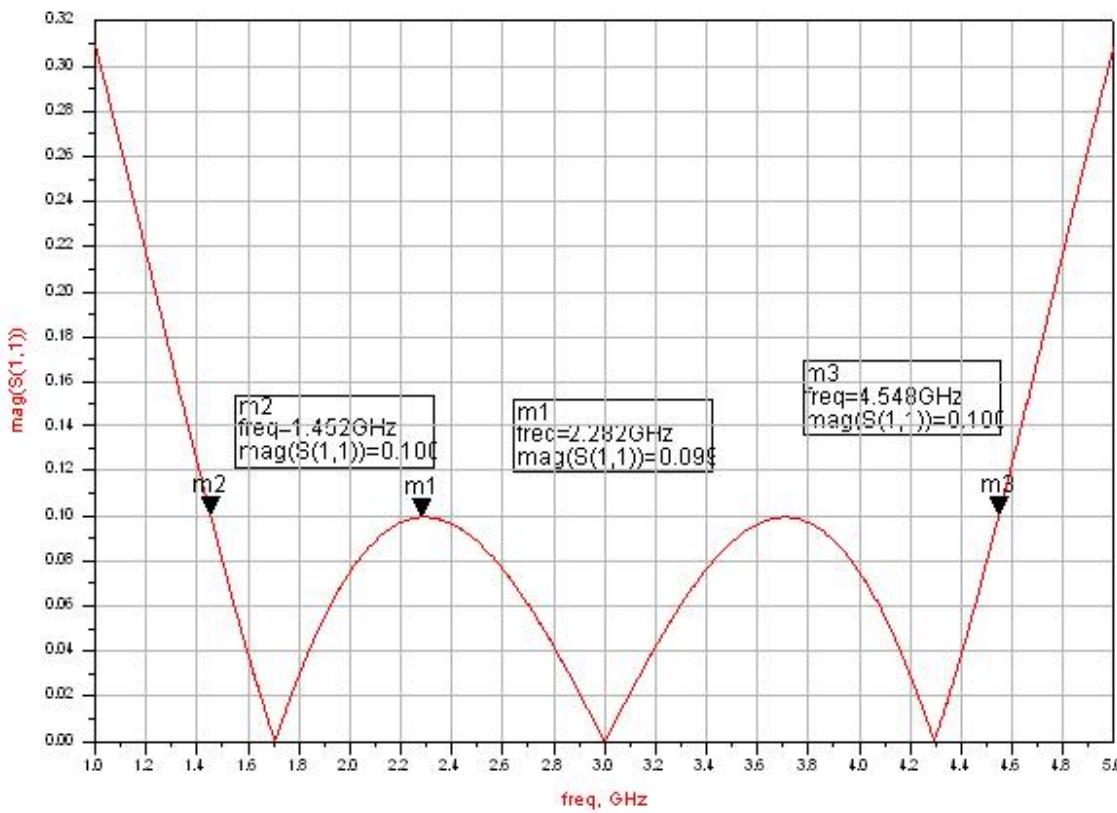
# Exemple

$$\frac{\Delta f}{f_0} = \frac{2(f_0 - f_m)}{f_0} = 2 - \frac{4\theta_m}{\pi} = 2 - \frac{4 \cdot 42.76^\circ}{180^\circ} = 1.045$$

$$\Delta f = 3.15 GHz$$

# Simultion

- Similarly Lab. 1



$$\Delta f = 3.096 \text{ GHz}$$

$$|\Gamma(3 \text{ GHz})| = 4.17 \cdot 10^{-5}$$

$$|\Gamma(2.282 \text{ GHz})| = 0.09925$$

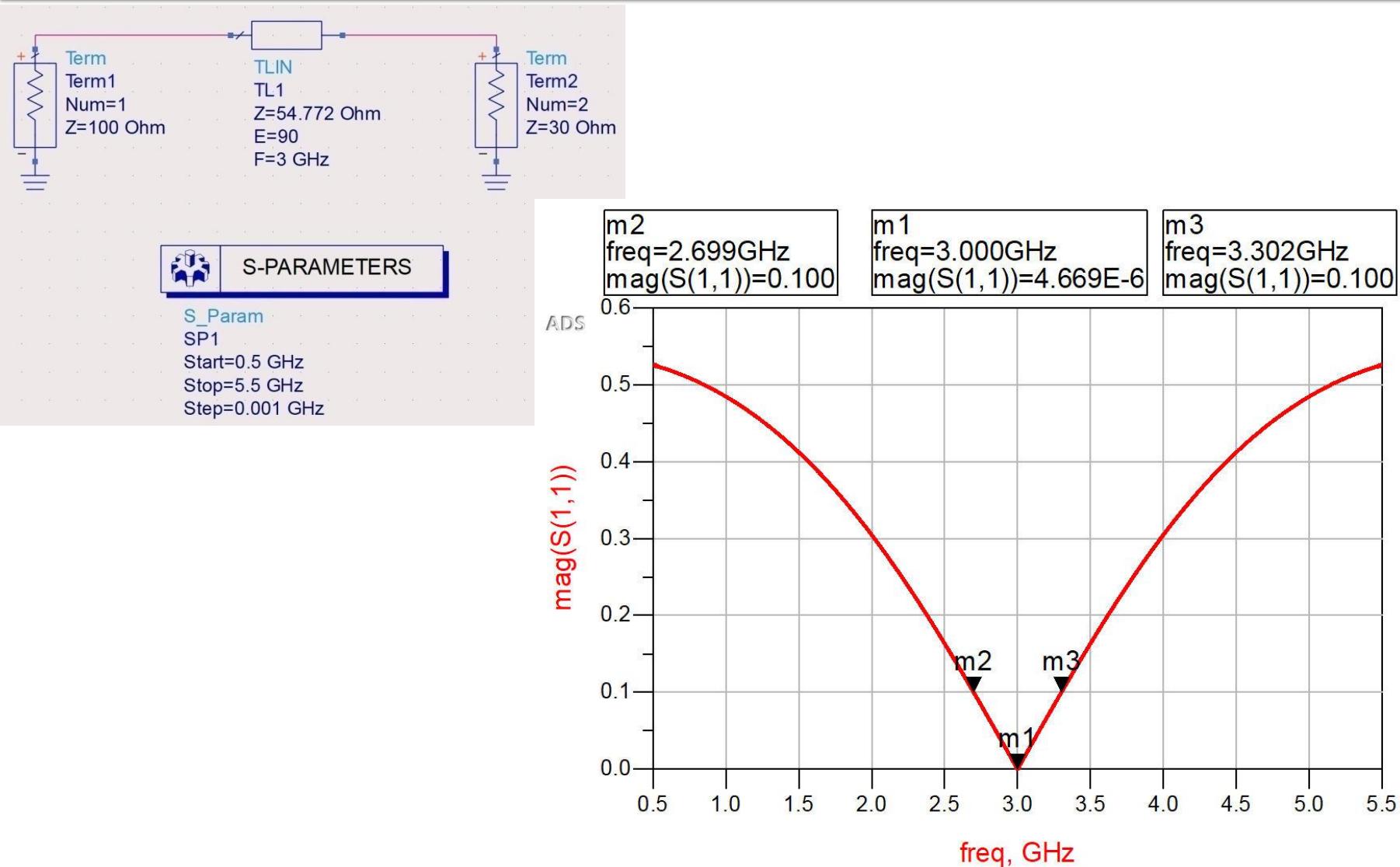
# Exact solutions

- G. L. Matthaei, L. Young, and E. M. T. Jones,  
*Microwave Filters, Impedance-Matching Networks, and Coupling Structures*, Artech House Books, Dedham, Mass. 1980

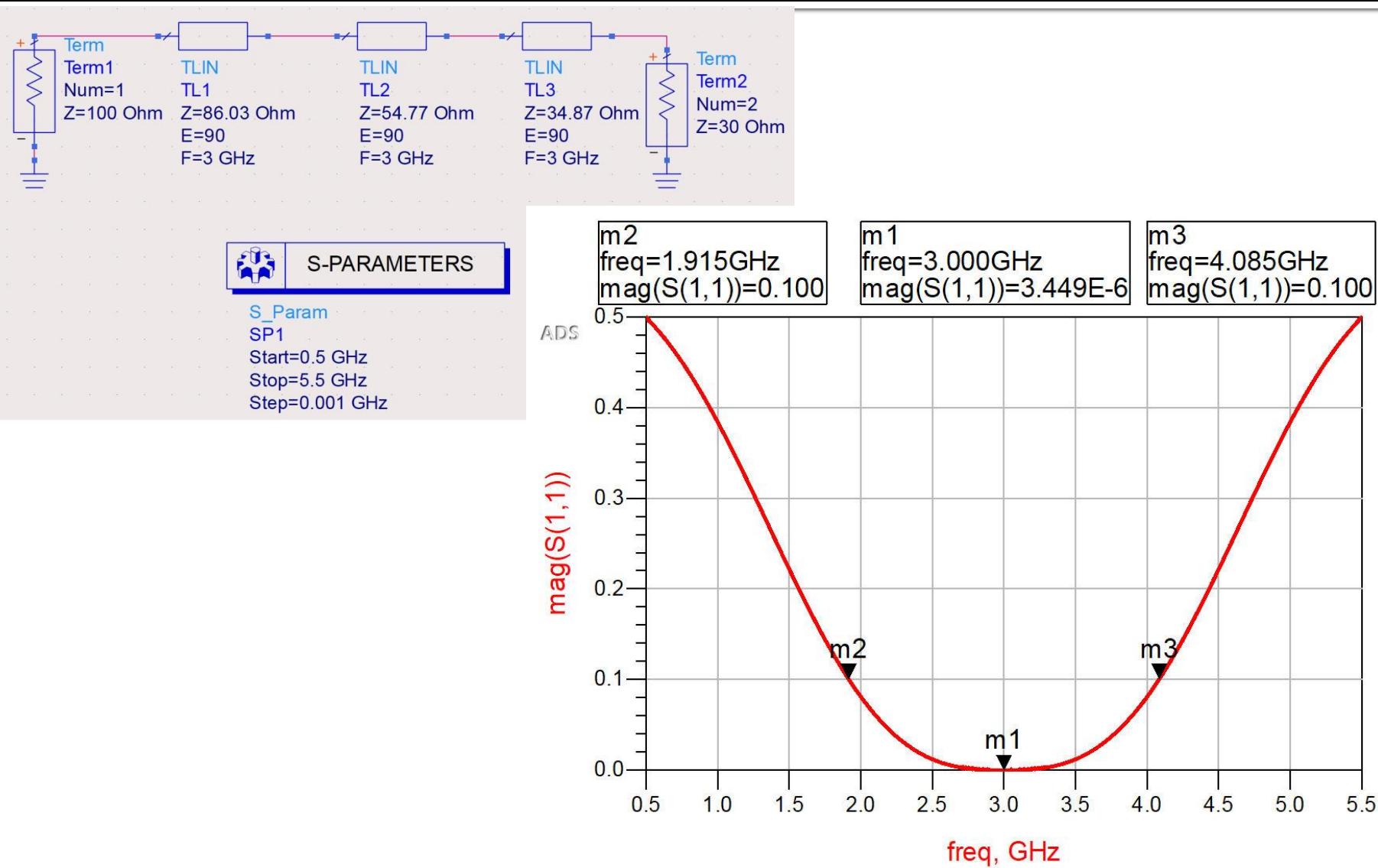
Laboratory 1

# Impedance Matching

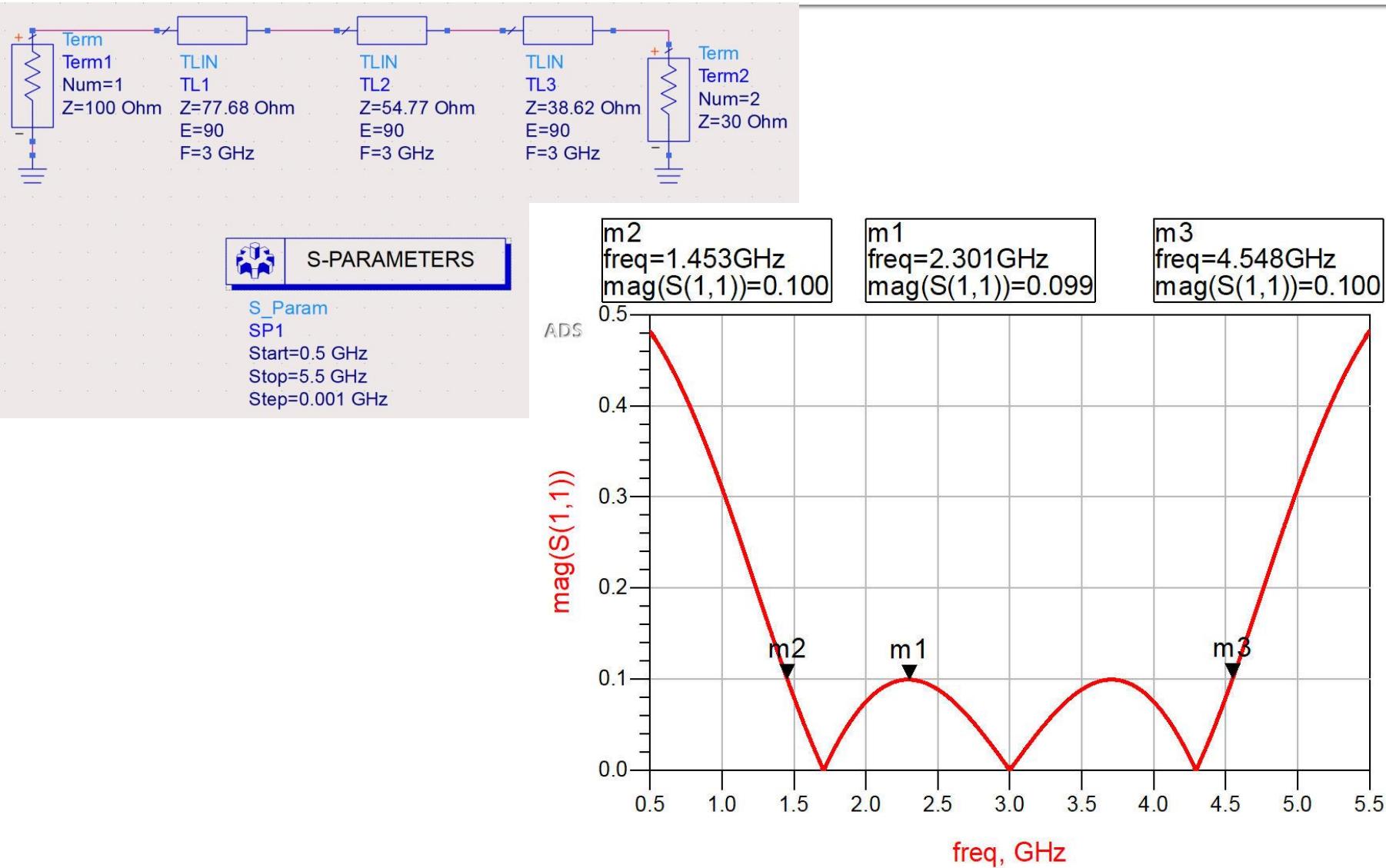
# The quarter-wave transformer



# Binomial multisection transformer



# Chebyshev multisection transformer



# Contact

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